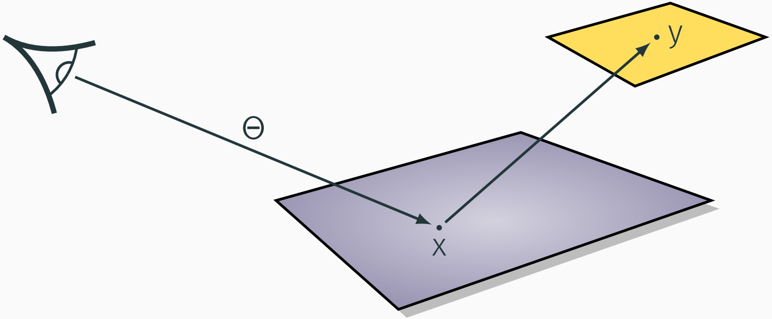
Line Sampling for Direct Illumination

Niels Billen Philip Dutré

24 June 2016

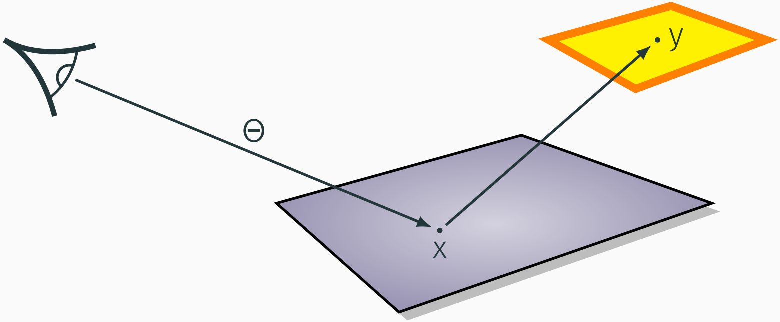
KU Leuven University

Direct Illumination



$$L_{\mathrm{direct}}\left(x \to \Theta\right) = \int_{\mathrm{A}} L\left(y \to x\right) f_{\mathrm{r}}\left(x, \overrightarrow{yx} \leftrightarrow \Theta\right) G\left(x, y\right) V\left(x, y\right) \mathrm{d}A$$

Direct Illumination



$$L_{\mathrm{direct}}\left(x \rightarrow \Theta\right) = \int_{A} \underbrace{L\left(y \rightarrow x\right)}_{\text{emission}} f_r\left(x, \overrightarrow{yx} \leftrightarrow \Theta\right) G\left(x, y\right) V\left(x, y\right) \mathrm{d}A$$

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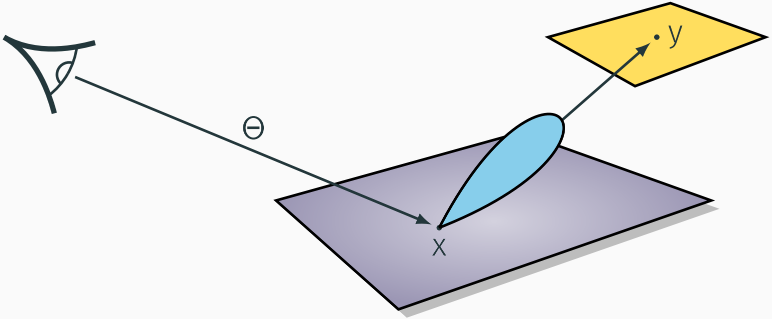

$$L_{\mathrm{direct}}\left(x \to \Theta\right) = \int_{\mathrm{A}} L\left(y \to x\right) \underbrace{f_{\mathrm{r}}\left(x, \overrightarrow{yx} \leftrightarrow \Theta\right)}_{\text{reflection}} G\left(x, y\right) V\left(x, y\right) \mathrm{d}A$$

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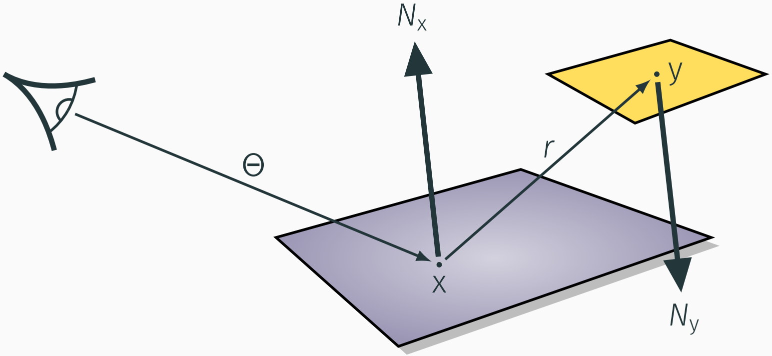
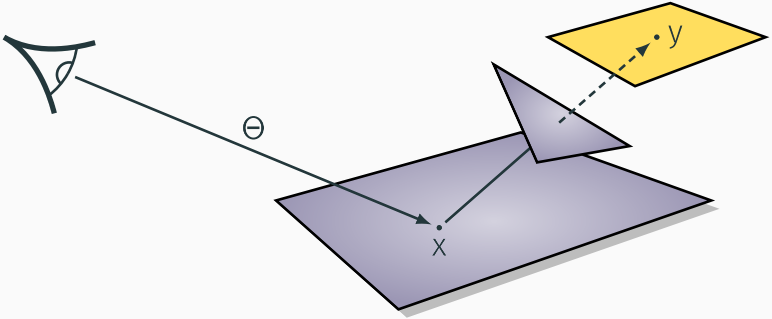


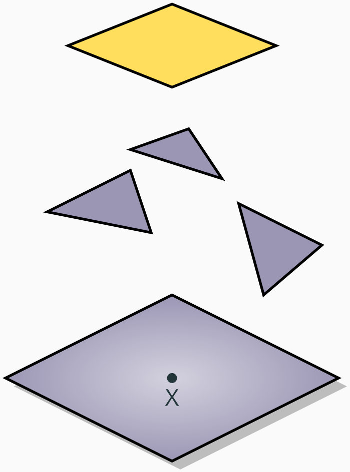
$$L_{\mathrm{direct}}\left(x \to \Theta\right) = \int_{A} L\left(y \to x\right) f_{r}\left(x, \overrightarrow{yx} \leftrightarrow \Theta\right) \underbrace{G\left(x, y\right)}_{\text{form factor}} V\left(x, y\right) \mathrm{d}A$$

Direct Illumination



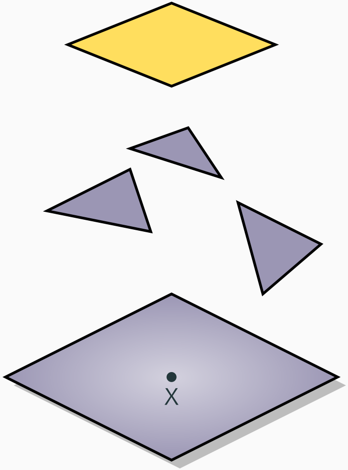
$$L_{\mathrm{direct}}\left(x \rightarrow \Theta\right) = \int_{\mathrm{A}} L\left(y \rightarrow x\right) f_{\mathrm{r}}\left(x, \overrightarrow{yx} \leftrightarrow \Theta\right) G\left(x, y\right) \underbrace{V\left(x, y\right)}_{\text{visibility}} \mathrm{d}A$$

Related Work



Related Work

- **Analytical evaluation:**
  - exact analytical evaluation
  - difficult visibility evaluation



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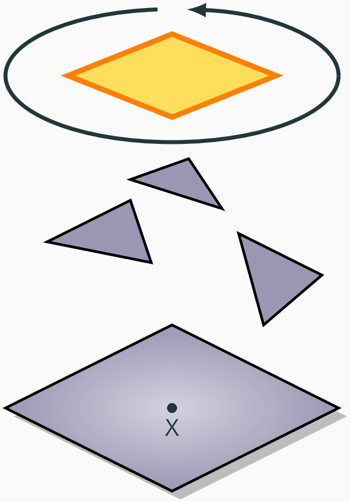

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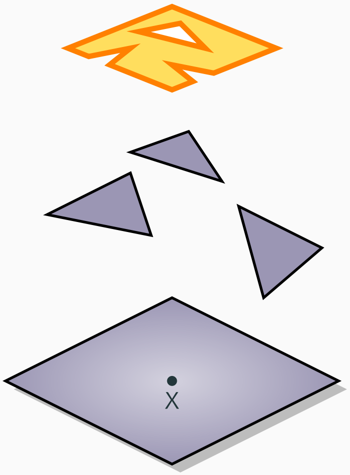

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- **Analytical evaluation:**
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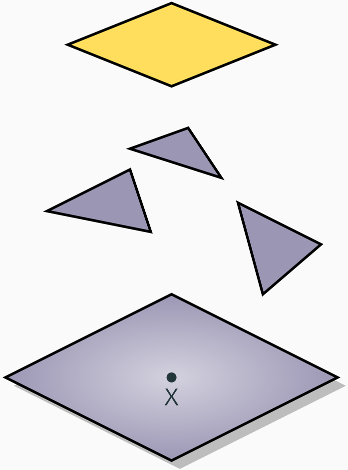
- **Stochastic evaluation:**
    - (quasi) random point sampling



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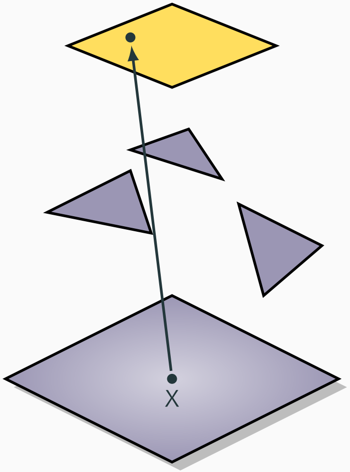

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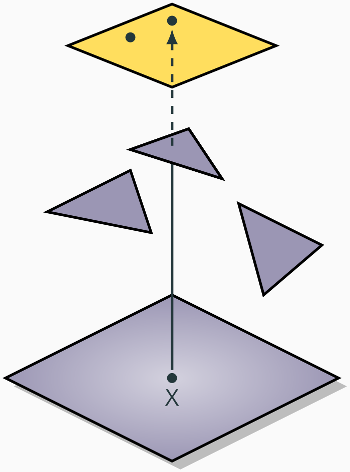

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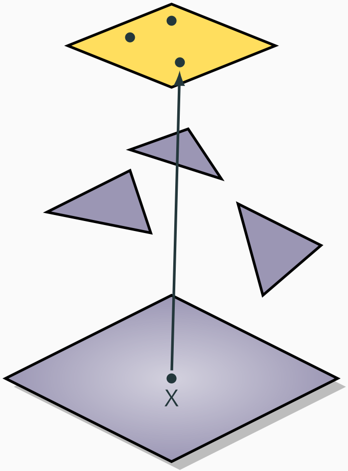

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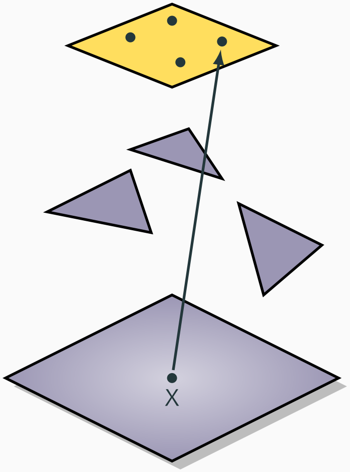

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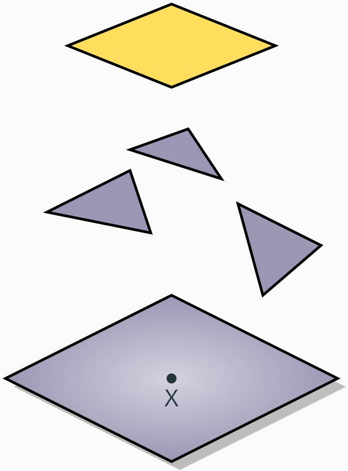

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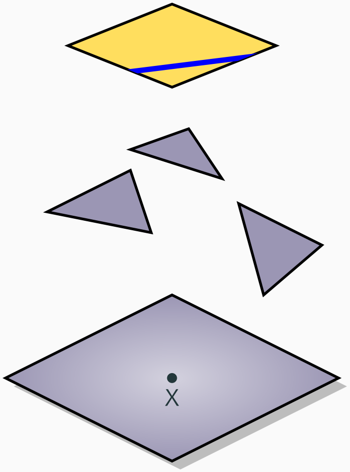

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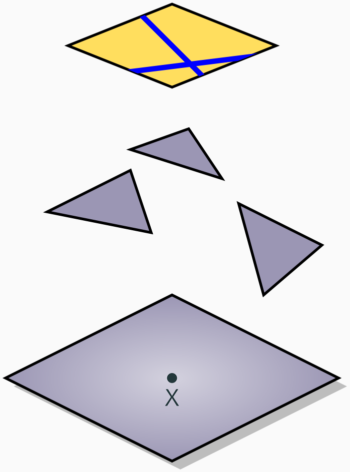

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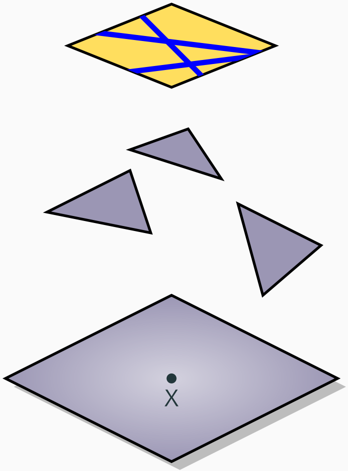

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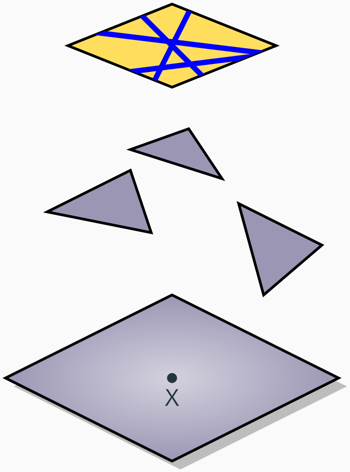

Line sample — generation

How to generate the line samples?

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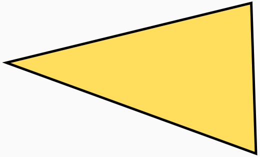
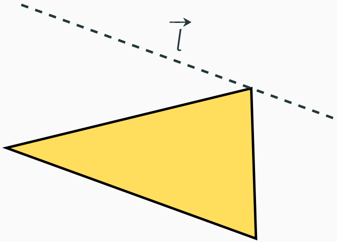


Line sample — generation

How to generate the line samples?

- choose a random direction $\vec{l}$



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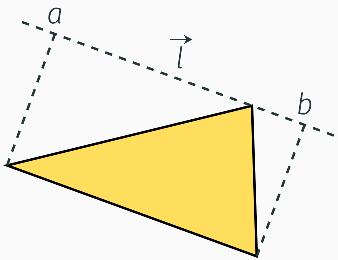

Line sample — generation

How to generate the line samples?

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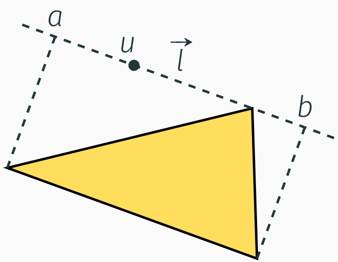

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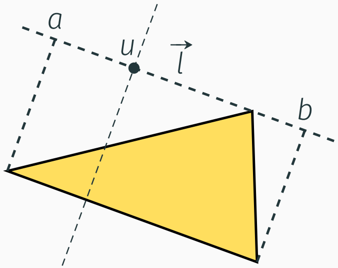

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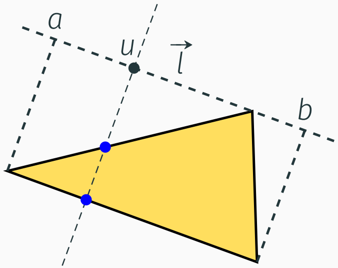

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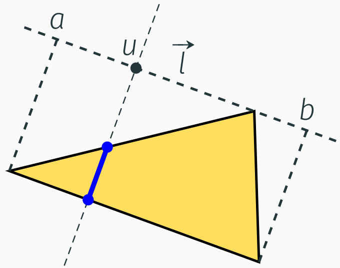

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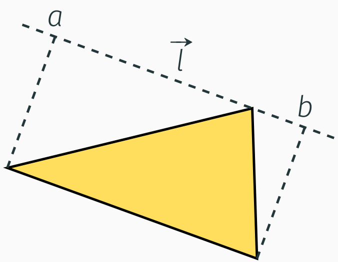

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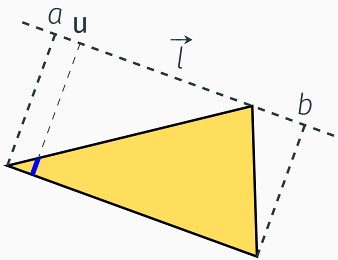

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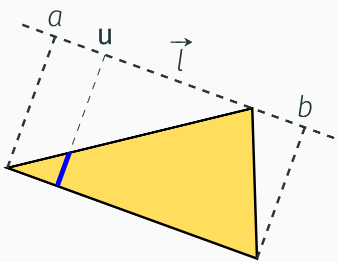

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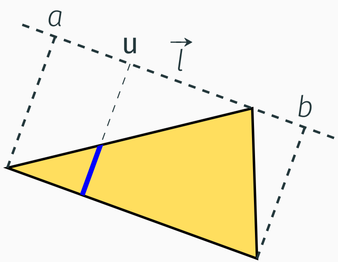

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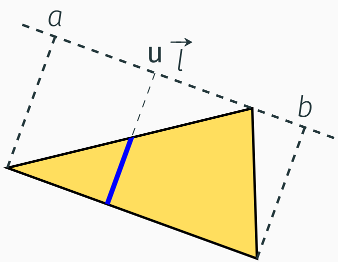

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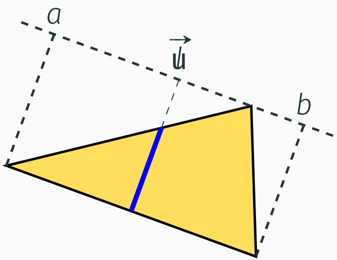

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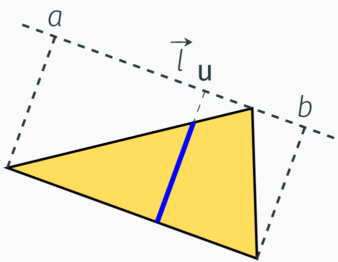

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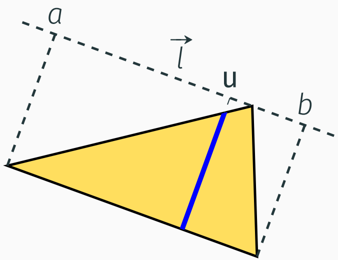

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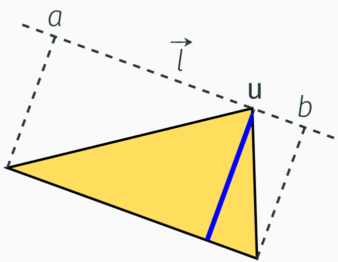

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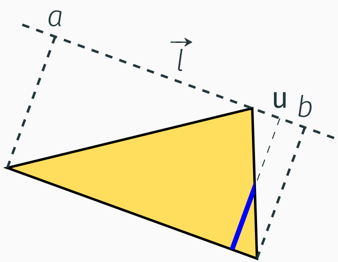

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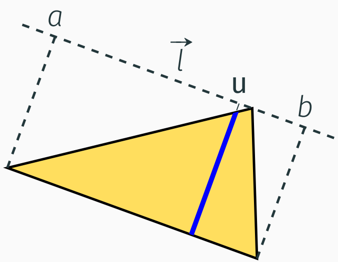

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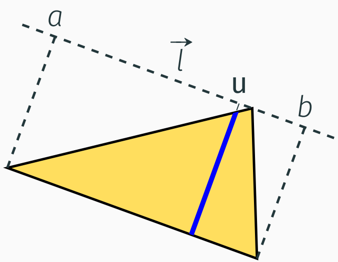

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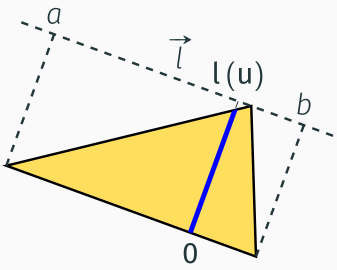
$$L_{\text{direct}}(\ldots) = \int_{\mathrm{A}} L(\ldots) f_{\mathrm{r}}(\ldots) G(\ldots) V(\ldots) \, \mathrm{d}A$$

$$= \int_a^b \int_0^{l(u)} L(\ldots) f_{\mathrm{r}}(\ldots) G(\ldots) V(\ldots) \, \mathrm{d}l \mathrm{d}u$$



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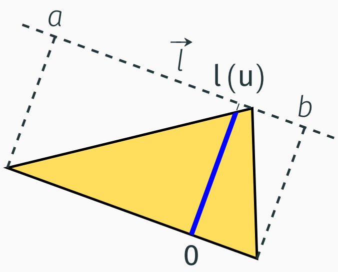

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\end{aligned}
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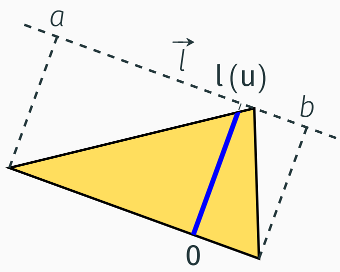

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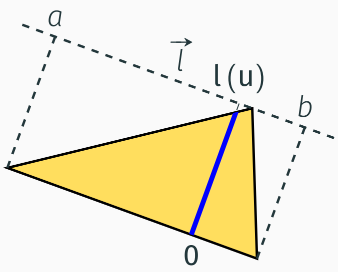
$$= \int_a^b L_{\mathrm{line}}(\ldots) \, \mathrm{d}u$$



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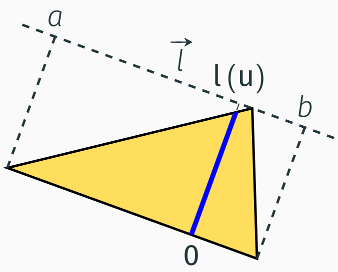
&= \underbrace{\int_a^b L_{\mathrm{line}}(\ldots)\, \mathrm{d}u}_{\text{stochastic evaluation}}
\end{aligned}
$$



Line sample — evaluation

Contribution of a line sample:

$$L_{\mathrm{line}}(\ldots) = \int_0^l L_{\mathsf{light}}(\ldots)\, f_{\mathrm{r}}(\ldots)\, G(\ldots)\, V(\ldots)\, \mathrm{d}t$$

Line sample — evaluation

Contribution of a line sample:

$$
\begin{aligned}
L_{\mathrm{line}}(\ldots) &= \int_0^l L_{\text{light}}(\ldots)\, f_{\mathrm{r}}(\ldots)\, G(\ldots)\, V(\ldots)\,\mathrm{d}t \\
&= L_{\text{light}} \int_0^l f_{\mathrm{r}}(\ldots)\, G(\ldots)\, V(\ldots)\,\mathrm{d}t \qquad \textit{assume diffuse light source}
\end{aligned}
$$

Line sample — evaluation

Contribution of a line sample:

$$L_{\mathrm{line}}(\ldots) = \int_0^l L_{\mathrm{light}}(\ldots)\, f_{\mathrm{r}}(\ldots)\, G(\ldots)\, V(\ldots)\, \mathrm{d}t$$

$$= L_{\mathrm{light}} \int_0^l f_{\mathrm{r}}(\ldots)\, G(\ldots)\, V(\ldots)\, \mathrm{d}t \qquad \textit{assume diffuse light source}$$

$$= L_{\mathrm{light}} \int_0^l f_{\mathrm{r}}(\ldots)\, G(\ldots)\, \mathrm{d}t \qquad \textit{assume visible line sample}$$

Line sample — evaluation

Contribution of a line sample:

$$
\begin{aligned}
L_{\mathrm{line}}(\ldots) &= \int_0^l L_{\mathit{light}}(\ldots)\, f_{\mathrm{r}}(\ldots)\, G(\ldots)\, V(\ldots)\, \mathrm{d}t \\
&= L_{\mathit{light}} \int_0^l f_{\mathrm{r}}(\ldots)\, G(\ldots)\, V(\ldots)\, \mathrm{d}t && \textit{assume diffuse light source} \\
&= L_{\mathit{light}} \underbrace{\int_0^l f_{\mathrm{r}}(\ldots)\, G(\ldots)\, \mathrm{d}t}_{\text{analytically integrable}} && \textit{assume visible line sample}
\end{aligned}
$$

Line sample — evaluation

Contribution of a line sample:

$$
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L_{\text{line}}(\ldots) &= \int_0^l L_{\text{light}}(\ldots)\, f_{\mathrm{r}}(\ldots)\, G(\ldots)\, V(\ldots)\,\mathrm{d}t \\
&= L_{\text{light}} \int_0^l f_{\mathrm{r}}(\ldots)\, G(\ldots)\, V(\ldots)\,\mathrm{d}t && \textit{assume diffuse light source} \\
&= L_{\text{light}} \underbrace{\int_0^l f_{\mathrm{r}}(\ldots)\, G(\ldots)\,\mathrm{d}t}_{\text{analytically integrable}} && \textit{assume visible line sample}
\end{aligned}
$$

Materials:

Line sample — evaluation

Contribution of a line sample:

$$
\begin{aligned}
L_{\mathrm{line}}(\ldots) &= \int_0^l L_{\mathrm{light}}(\ldots)\, f_{\mathrm{r}}(\ldots)\, G(\ldots)\, V(\ldots)\, \mathrm{d}t \\
&= L_{\mathrm{light}} \int_0^l f_{\mathrm{r}}(\ldots)\, G(\ldots)\, V(\ldots)\, \mathrm{d}t && \textit{assume diffuse light source} \\
&= L_{\mathrm{light}} \underbrace{\int_0^l f_{\mathrm{r}}(\ldots)\, G(\ldots)\, \mathrm{d}t}_{\text{analytically integrable}} && \textit{assume visible line sample}
\end{aligned}
$$

Materials:

- Diffuse BRDF
- Phong BRDF

Diffuse material

Diffuse BRDF:

$$f_{\mathrm{r}}\left(x, \overrightarrow{yx} \leftrightarrow \Theta\right) = \frac{k_{\mathrm{d}}}{\pi}$$

Diffuse material

Diffuse BRDF:

$$f_{\mathrm{r}}\left(x, \overrightarrow{yx} \leftrightarrow \Theta\right) = \frac{k_{\mathrm{d}}}{\pi}$$

Line sample contribution (extension of [Nishita et al., 1985]):

$$L_{\mathrm{line}}\left(\ldots\right) = L_{\mathrm{light}} \frac{k_{\mathrm{d}}}{\pi} \int_0^l G\left(\ldots\right) \mathrm{d}t$$

Diffuse material

Diffuse BRDF:

$$f_{\mathrm{r}}\left(x, \overrightarrow{yx} \leftrightarrow \Theta\right) = \frac{k_{\mathrm{d}}}{\pi}$$

Line sample contribution (extension of [Nishita et al., 1985]):

$$\begin{aligned} L_{\mathrm{line}}(\ldots) &= L_{\mathrm{light}} \frac{k_{\mathrm{d}}}{\pi} \int_0^l G(\ldots)\, \mathrm{d}t \\ &= L_{\mathrm{light}} \frac{k_{\mathrm{d}}}{2\pi} \left(\frac{(A - BD)}{(C - D^2)^{\frac{3}{2}}} \left(\tan^{-1}\left(\frac{D}{\sqrt{C - D^2}} \right) - \tan^{-1}\left(\frac{D + l}{\sqrt{C - D^2}} \right) \right) \right) \\ &\quad - L_{\mathrm{light}} \frac{k_{\mathrm{d}}}{2\pi} \frac{lF\left(BC\left(C + l\right)\right) + A\left(C - Dl - 2D^2\right)}{C\left(C - D^2\right)\left(l^2 + 2Dl + C\right)} \end{aligned}$$

Phong material

Phong BRDF:

$$f_r\left(x, \overrightarrow{yx} \leftrightarrow \Theta\right) = k_s \frac{(n+2)\left(\overrightarrow{yx} \cdot \overrightarrow{R}\right)}{2\pi}$$

Phong material

Phong BRDF:

$$f_{\mathrm{r}}\left(x, \overrightarrow{yx} \leftrightarrow \Theta\right) = k_{\mathrm{s}} \frac{(n+2)\left(\overrightarrow{yx} \cdot \vec{R}\right)}{2\pi}$$

Line sample contribution (extension of [Poulin and Amanatides, 1991]):

$$L_{\mathrm{line}}\left(\ldots\right) = L_{\mathrm{light}} k_{\mathrm{s}} \frac{n+2}{2\pi} \int_0^l G\left(\ldots\right)\left(\overrightarrow{yx} \cdot \vec{R}\right) \mathrm{d}t$$

Phong material

Phong BRDF:

$$f_{\mathrm{r}}\left(x, \overrightarrow{yx} \leftrightarrow \Theta\right) = k_{\mathrm{s}} \frac{(n+2)\left(\overrightarrow{yx} \cdot \vec{R}\right)}{2\pi}$$

Line sample contribution (extension of [Poulin and Amanatides, 1991]):

$$\begin{aligned} L_{\mathrm{line}}(\ldots) &= L_{\mathrm{light}} k_{\mathrm{s}} \frac{n+2}{2\pi} \int_0^l G(\ldots)\left(\overrightarrow{yx} \cdot \vec{R}\right) \mathrm{d}t \\ &= L_{\mathrm{light}} k_{\mathrm{s}} \frac{n+2}{2\pi} \frac{\sin\left(\varphi_{\overrightarrow{N_{\mathrm{x}}}}\right) \sin\left(\varphi_{\overrightarrow{N_{\mathrm{y}}}}\right)}{\overrightarrow{L_{\mathrm{o_x}}} \sin\left(\Theta_{\mathrm{L}}\right) - \overrightarrow{L_{\mathrm{o_y}}} \sin\left(\Theta_{\mathrm{L}}\right)} \cdot \\ &\quad \int_{\theta_{\mathrm{min}}}^{\theta_{\mathrm{max}}} \cos\left(\theta - \theta_{\overrightarrow{N_{\mathrm{x}}}}\right) \cos\left(\theta - \theta_{\overrightarrow{N_{\mathrm{y}}}}\right) \cos\left(\theta - \theta_{\vec{R}}\right)^n \mathrm{d}\theta \end{aligned}$$

Phong material

Phong BRDF:

$$f_{\mathrm{r}}\left(x, \overrightarrow{yx} \leftrightarrow \Theta\right) = k_{\mathrm{s}} \frac{(n+2)\left(\overrightarrow{yx} \cdot \vec{R}\right)}{2\pi}$$

Line sample contribution (extension of [Poulin and Amanatides, 1991]):

$$L_{\mathrm{line}}(\ldots) = L_{\mathrm{light}} k_{\mathrm{s}} \frac{n+2}{2\pi} \int_0^l G(\ldots)\left(\overrightarrow{yx} \cdot \vec{R}\right) \mathrm{d}t$$

$$= L_{\mathrm{light}} k_{\mathrm{s}} \frac{n+2}{2\pi} \frac{\sin\left(\varphi_{\overrightarrow{N_{\mathrm{x}}}}\right) \sin\left(\varphi_{\overrightarrow{N_{\mathrm{y}}}}\right)}{\overrightarrow{L_{\mathrm{o_x}}} \sin\left(\Theta_{\mathrm{L}}\right) - \overrightarrow{L_{\mathrm{o_y}}} \sin\left(\Theta_{\mathrm{L}}\right)} \cdot \underbrace{\int_{\theta_{\min}}^{\theta_{\max}} \cos\left(\theta - \theta_{\overrightarrow{N_{\mathrm{x}}}}\right) \cos\left(\theta - \theta_{\overrightarrow{N_{\mathrm{y}}}}\right) \cos\left(\theta - \theta_{\vec{R}}\right)^n \mathrm{d}\theta}_{\text{integral over the angle spanned by the line sample}}$$

Phong material

Line sample contribution:

$$L_{\text{line}}(\ldots) = \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \cos\left(\theta - \theta_{\overrightarrow{N_{\mathrm{x}}}}\right) \cos\left(\theta - \theta_{\overrightarrow{N_{\mathrm{y}}}}\right) \cos\left(\theta - \theta_{\overrightarrow{R}}\right)^{n} \mathrm{d}\theta$$

Phong material

Line sample contribution:

$$
\begin{aligned}
L_{\mathrm{line}}(\ldots) &= \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \cos\left(\theta - \theta_{\overrightarrow{N_{\mathrm{x}}}}\right) \cos\left(\theta - \theta_{\overrightarrow{N_{\mathrm{y}}}}\right) \cos\left(\theta - \theta_{\vec{R}}\right)^n \,\mathrm{d}\theta \\
&= -\cos\left(\theta_{\vec{R}} - \theta_{\overrightarrow{N_{\mathrm{x}}}}\right) \sin\left(\theta_{\vec{R}} - \theta_{\overrightarrow{N_{\mathrm{y}}}}\right) \int_{\theta_{\text{min}}-\theta_{\vec{R}}}^{\theta_{\text{max}}-\theta_{\vec{R}}} \cos(u)^{n+1} \sin(u) \,\mathrm{d}u \\
&\quad - \sin\left(\theta_{\vec{R}} - \theta_{\overrightarrow{N_{\mathrm{x}}}}\right) \cos\left(\theta_{\vec{R}} - \theta_{\overrightarrow{N_{\mathrm{y}}}}\right) \int_{\theta_{\text{min}}-\theta_{\vec{R}}}^{\theta_{\text{max}}-\theta_{\vec{R}}} \cos(u)^{n+1} \sin(u) \,\mathrm{d}u \\
&\quad + \cos\left(\theta_{\vec{R}} - \theta_{\overrightarrow{N_{\mathrm{x}}}}\right) \cos\left(\theta_{\vec{R}} - \theta_{\overrightarrow{N_{\mathrm{y}}}}\right) \int_{\theta_{\text{min}}-\theta_{\vec{R}}}^{\theta_{\text{max}}-\theta_{\vec{R}}} \cos(u)^{n+2} \,\mathrm{d}u \\
&\quad + \sin\left(\theta_{\vec{R}} - \theta_{\overrightarrow{N_{\mathrm{x}}}}\right) \sin\left(\theta_{\vec{R}} - \theta_{\overrightarrow{N_{\mathrm{y}}}}\right) \int_{\theta_{\text{min}}-\theta_{\vec{R}}}^{\theta_{\text{max}}-\theta_{\vec{R}}} \cos(u)^{n} \sin(u)^2 \,\mathrm{d}u
\end{aligned}
$$

Phong material

Line sample contribution:

$$
\begin{aligned}
L_{\mathrm{line}}(\dots) &= \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \cos\left(\theta - \theta_{\vec{N_{\mathrm{x}}}}\right) \cos\left(\theta - \theta_{\vec{N_{\mathrm{y}}}}\right) \cos\left(\theta - \theta_{\vec{R}}\right)^n \, \mathrm{d}\theta \\
&= -\cos\left(\theta_{\vec{R}} - \theta_{\vec{N_{\mathrm{x}}}}\right) \sin\left(\theta_{\vec{R}} - \theta_{\vec{N_{\mathrm{y}}}}\right) \int_{\theta_{\text{min}}-\theta_{\vec{R}}}^{\theta_{\text{max}}-\theta_{\vec{R}}} \cos(u)^{n+1} \sin(u) \, \mathrm{d}u \\
&\quad - \sin\left(\theta_{\vec{R}} - \theta_{\vec{N_{\mathrm{x}}}}\right) \cos\left(\theta_{\vec{R}} - \theta_{\vec{N_{\mathrm{y}}}}\right) \int_{\theta_{\text{min}}-\theta_{\vec{R}}}^{\theta_{\text{max}}-\theta_{\vec{R}}} \cos(u)^{n+1} \sin(u) \, \mathrm{d}u \\
&\quad + \cos\left(\theta_{\vec{R}} - \theta_{\vec{N_{\mathrm{x}}}}\right) \cos\left(\theta_{\vec{R}} - \theta_{\vec{N_{\mathrm{y}}}}\right) \int_{\theta_{\text{min}}-\theta_{\vec{R}}}^{\theta_{\text{max}}-\theta_{\vec{R}}} \cos(u)^{n+2} \, \mathrm{d}u \\
&\quad + \sin\left(\theta_{\vec{R}} - \theta_{\vec{N_{\mathrm{x}}}}\right) \sin\left(\theta_{\vec{R}} - \theta_{\vec{N_{\mathrm{y}}}}\right) \int_{\theta_{\text{min}}-\theta_{\vec{R}}}^{\theta_{\text{max}}-\theta_{\vec{R}}} \cos(u)^{n} \sin(u)^2 \, \mathrm{d}u
\end{aligned}
$$

Integral identities:

$$
\int \cos(\theta) \sin(\theta) \, \mathrm{d}\theta = \frac{-\cos(\theta)^{n+1}}{n+1}
$$

Phong material

Line sample contribution:

$$
\begin{aligned}
L_{\text{line}}(\ldots) &= \int_{\theta_{\min}}^{\theta_{\max}} \cos\left(\theta - \theta_{\vec{N_x}}\right) \cos\left(\theta - \theta_{\vec{N_y}}\right) \cos\left(\theta - \theta_{\vec{R}}\right)^n \mathrm{d}\theta \\
&= -\cos\left(\theta_{\vec{R}} - \theta_{\vec{N_x}}\right) \sin\left(\theta_{\vec{R}} - \theta_{\vec{N_y}}\right) \int_{\theta_{\min}-\theta_{\vec{R}}}^{\theta_{\max}-\theta_{\vec{R}}} \cos(u)^{n+1} \sin(u)\, \mathrm{d}u \\
&\quad - \sin\left(\theta_{\vec{R}} - \theta_{\vec{N_x}}\right) \cos\left(\theta_{\vec{R}} - \theta_{\vec{N_y}}\right) \int_{\theta_{\min}-\theta_{\vec{R}}}^{\theta_{\max}-\theta_{\vec{R}}} \cos(u)^{n+1} \sin(u)\, \mathrm{d}u \\
&\quad + \cos\left(\theta_{\vec{R}} - \theta_{\vec{N_x}}\right) \cos\left(\theta_{\vec{R}} - \theta_{\vec{N_y}}\right) \int_{\theta_{\min}-\theta_{\vec{R}}}^{\theta_{\max}-\theta_{\vec{R}}} \cos(u)^{n+2}\, \mathrm{d}u \\
&\quad + \sin\left(\theta_{\vec{R}} - \theta_{\vec{N_x}}\right) \sin\left(\theta_{\vec{R}} - \theta_{\vec{N_y}}\right) \int_{\theta_{\min}-\theta_{\vec{R}}}^{\theta_{\max}-\theta_{\vec{R}}} \cos(u)^{n} \sin(u)^2\, \mathrm{d}u
\end{aligned}
$$

Integral identities:

$$
\int \cos(\theta) \sin(\theta)\, \mathrm{d}\theta = \frac{-\cos(\theta)^{n+1}}{n+1}
$$

$$
\int \cos(\theta)^n\, \mathrm{d}\theta = \frac{\cos(\theta)^{n-1} \sin(\theta)}{n} + \frac{n-1}{n} \int \cos(\theta)^{n-2}\, \mathrm{d}\theta
$$

Phong material

Line sample contribution:

$$
\begin{aligned}
L_{\text{line}}(\ldots) &= \int_{\theta_{\min}}^{\theta_{\max}} \cos\left(\theta - \theta_{\vec{N_x}}\right) \cos\left(\theta - \theta_{\vec{N_y}}\right) \cos\left(\theta - \theta_{\vec{R}}\right)^n \mathrm{d}\theta \\
&= -\cos\left(\theta_{\vec{R}} - \theta_{\vec{N_x}}\right) \sin\left(\theta_{\vec{R}} - \theta_{\vec{N_y}}\right) \int_{\theta_{\min}-\theta_{\vec{R}}}^{\theta_{\max}-\theta_{\vec{R}}} \cos(u)^{n+1} \sin(u)\, \mathrm{d}u \\
&\quad - \sin\left(\theta_{\vec{R}} - \theta_{\vec{N_x}}\right) \cos\left(\theta_{\vec{R}} - \theta_{\vec{N_y}}\right) \int_{\theta_{\min}-\theta_{\vec{R}}}^{\theta_{\max}-\theta_{\vec{R}}} \cos(u)^{n+1} \sin(u)\, \mathrm{d}u \\
&\quad + \cos\left(\theta_{\vec{R}} - \theta_{\vec{N_x}}\right) \cos\left(\theta_{\vec{R}} - \theta_{\vec{N_y}}\right) \int_{\theta_{\min}-\theta_{\vec{R}}}^{\theta_{\max}-\theta_{\vec{R}}} \cos(u)^{n+2}\, \mathrm{d}u \\
&\quad + \sin\left(\theta_{\vec{R}} - \theta_{\vec{N_x}}\right) \sin\left(\theta_{\vec{R}} - \theta_{\vec{N_y}}\right) \int_{\theta_{\min}-\theta_{\vec{R}}}^{\theta_{\max}-\theta_{\vec{R}}} \cos(u)^{n} \sin(u)^2\, \mathrm{d}u
\end{aligned}
$$

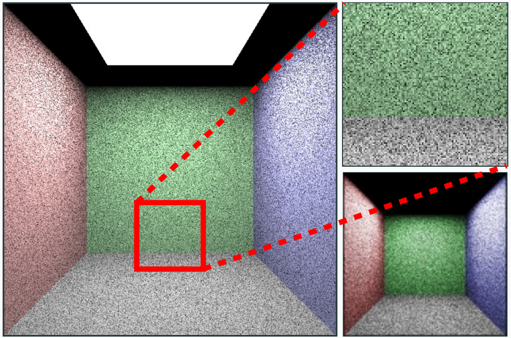
Integral identities:

$$\int \cos(\theta)\sin(\theta)\,\mathrm{d}\theta = \frac{-\cos(\theta)^{n+1}}{n+1} \qquad \int \cos(\theta)^n\,\mathrm{d}\theta = \frac{\cos(\theta)^{n-1}\sin(\theta)}{n} + \frac{n-1}{n}\int \cos(\theta)^{n-2}\,\mathrm{d}\theta$$

Line sampling — convergence

Cornell box

Inset



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Eurographics logo

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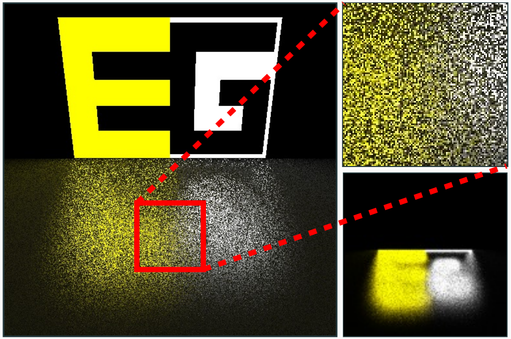
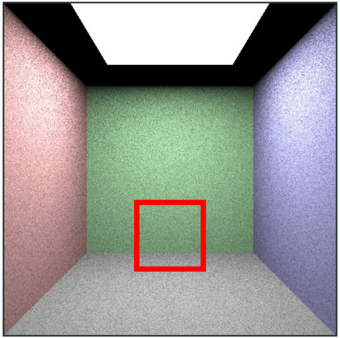


Difference

Line sampling — convergence

Cornell box



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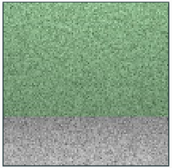

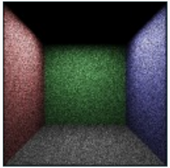

Difference

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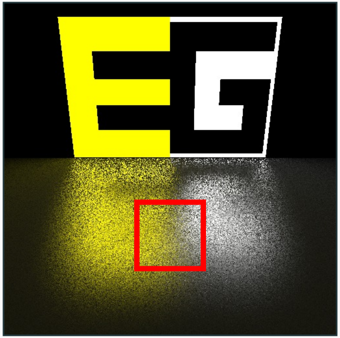

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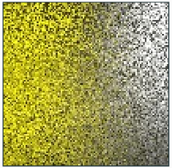
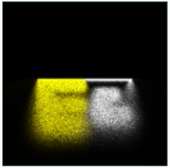
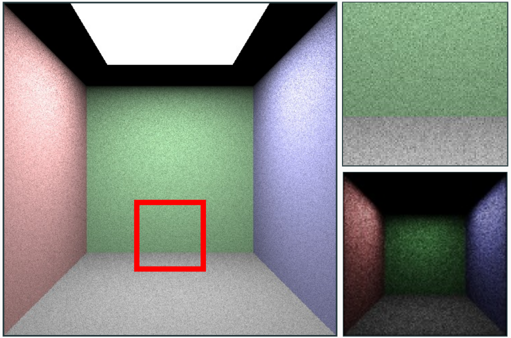




Difference

Line sampling — convergence

Cornell box — Inset



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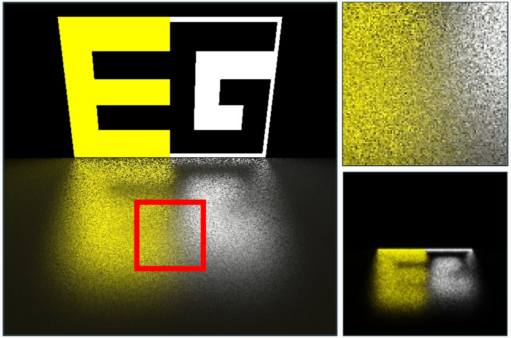
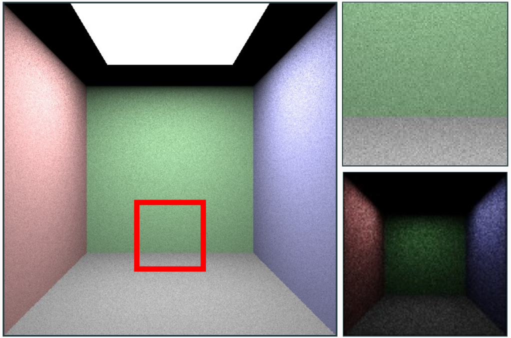


Difference

Line sampling — convergence

Cornell box | Inset



Difference

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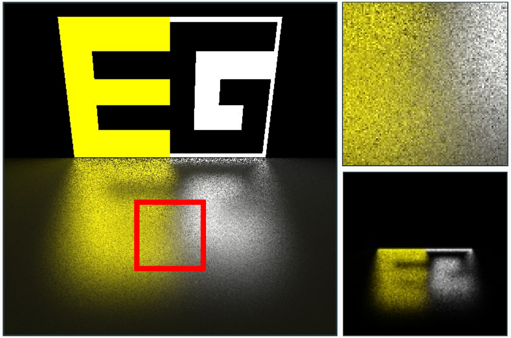
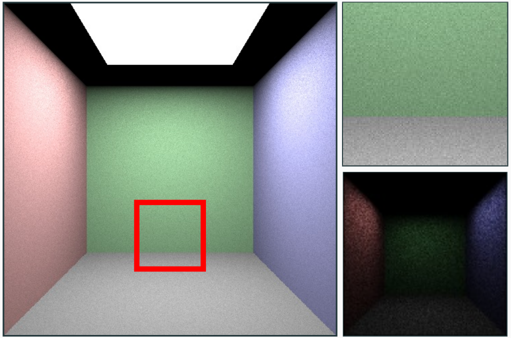


Difference

Line sampling — convergence

Cornell box

Inset



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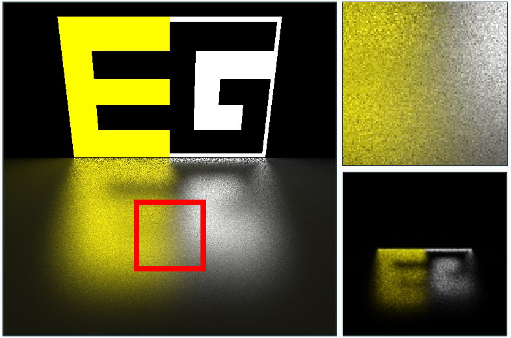
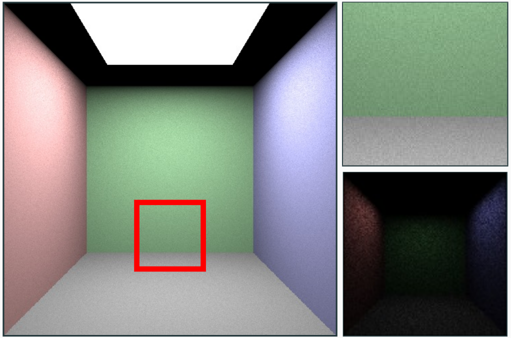


Difference

Line sampling — convergence

Cornell box

Inset



Difference

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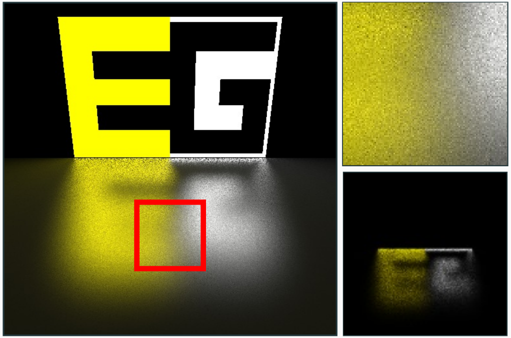
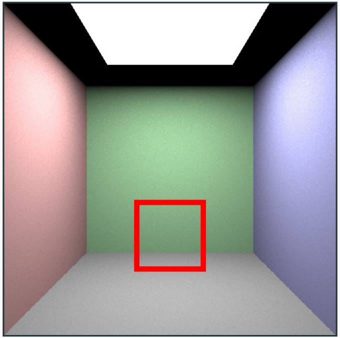


Difference

Line sampling — convergence

Cornell box



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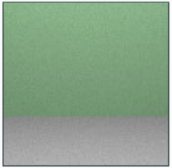

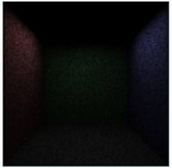

Difference

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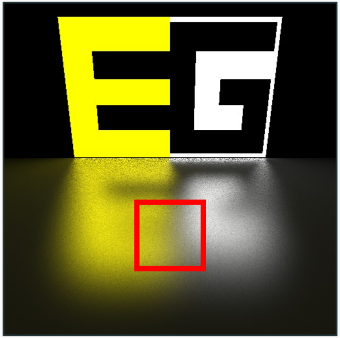

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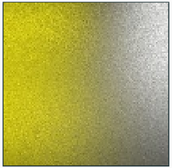
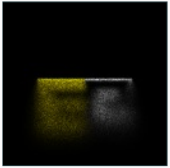
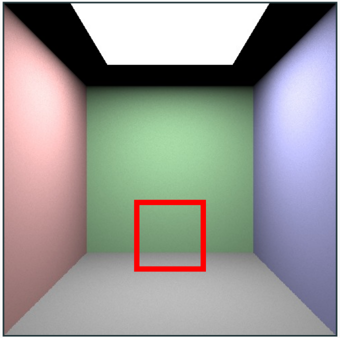




Difference

Line sampling — convergence

Cornell box



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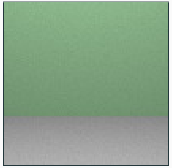
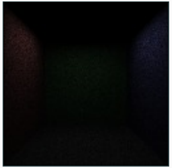
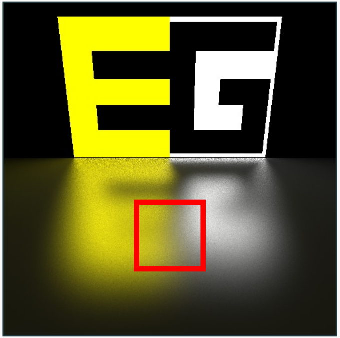




Difference

Eurographics logo



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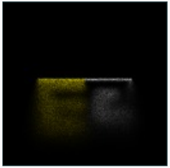
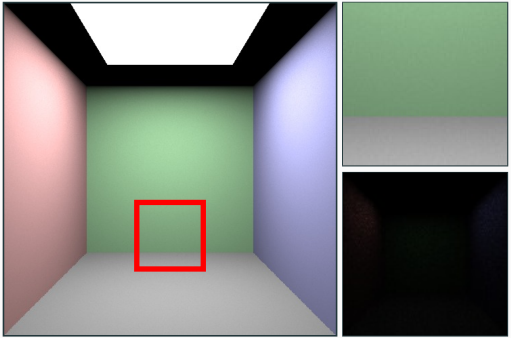


Difference

Line sampling — convergence

Cornell box

Inset



Difference

Eurographics logo

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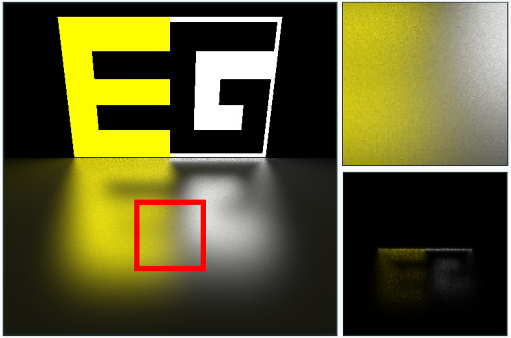
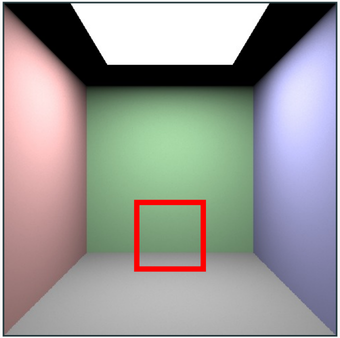


Difference

Line sampling — convergence

Cornell box



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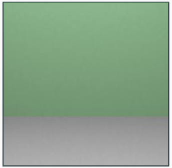
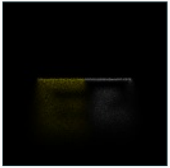


Difference

Eurographics logo

Inset

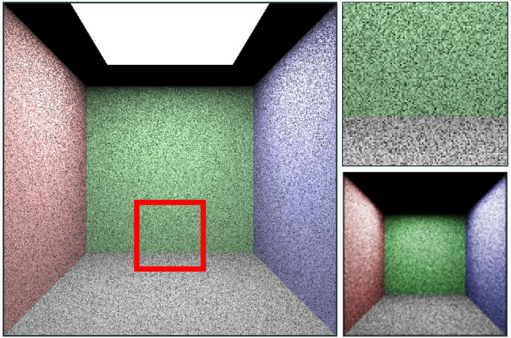


Difference

Line sampling — convergence

Cornell box

Inset



Difference

Eurographics logo

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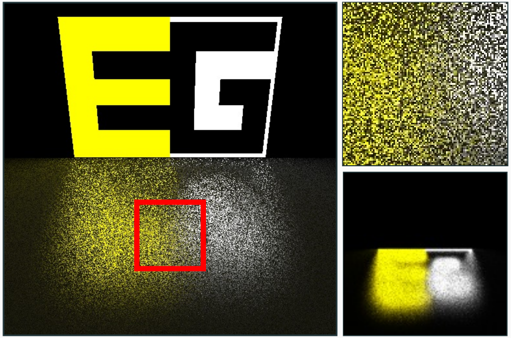
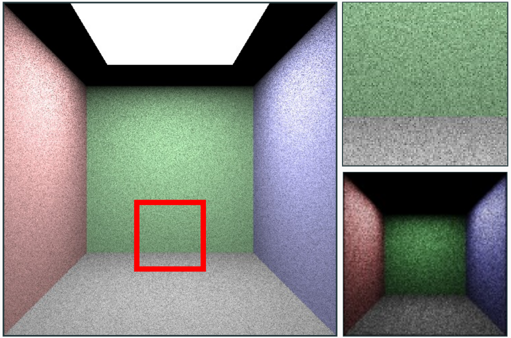


Difference

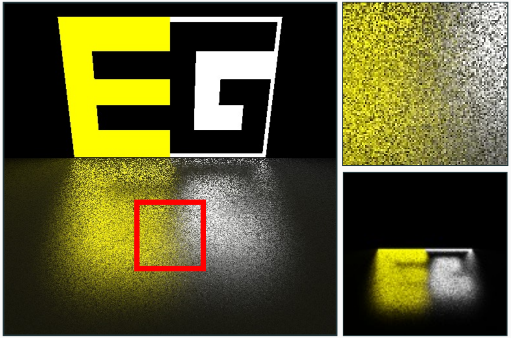
Line sampling — convergence

Cornell box | Inset



Difference

Eurographics logo | Inset

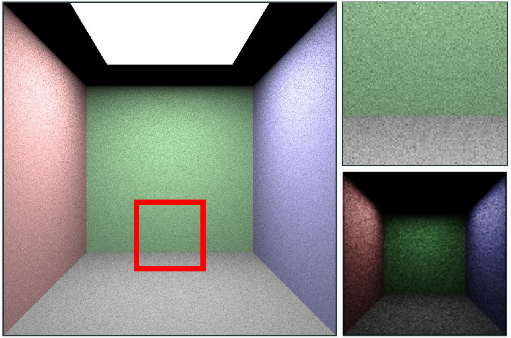


Difference

Line sampling — convergence

Cornell box

Inset



Difference

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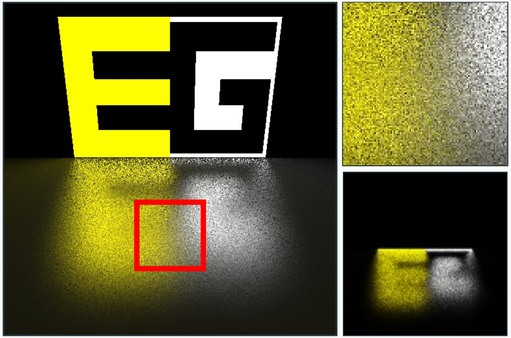
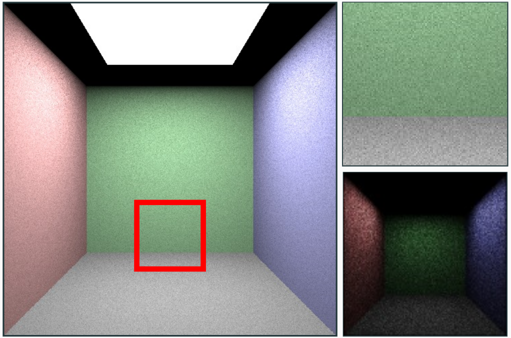


Difference

Line sampling — convergence

Cornell box

Inset



Difference

Eurographics logo

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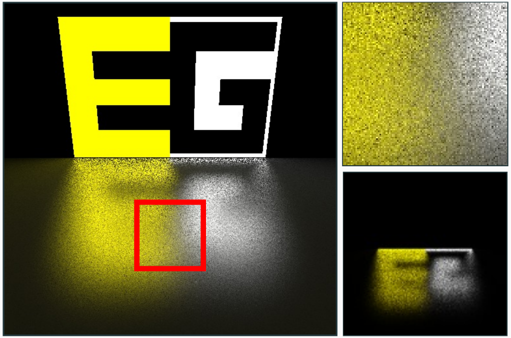
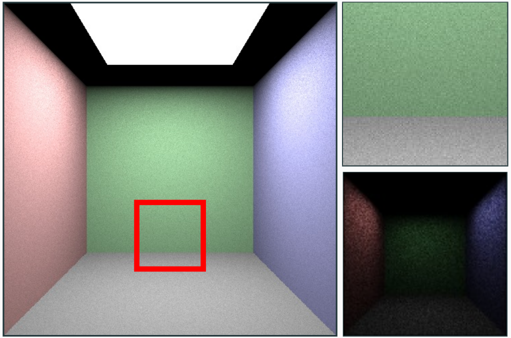


Difference

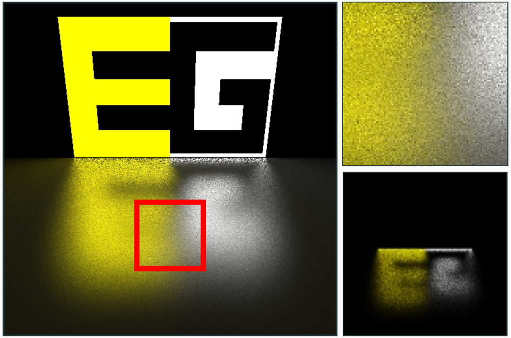
Line sampling — convergence

Cornell box Inset



Difference

Eurographics logo Inset



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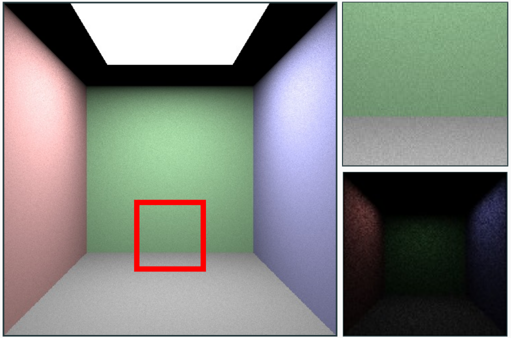

Difference

Eurographics logo

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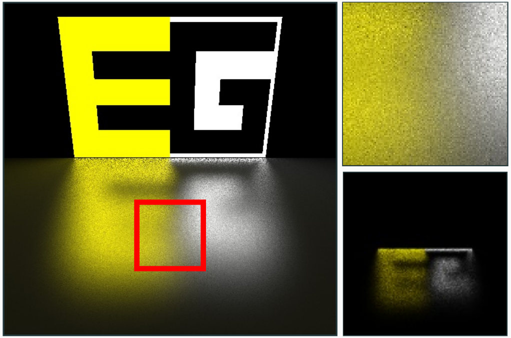

Difference

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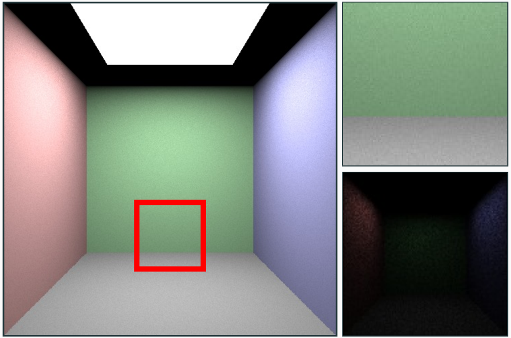
Inset



Difference

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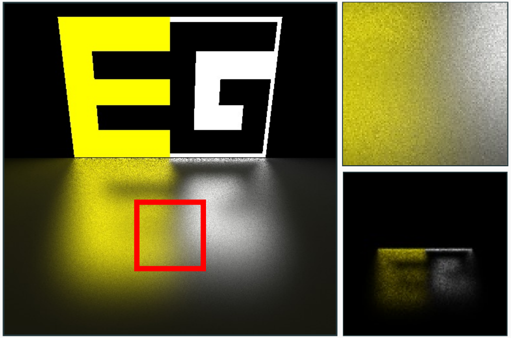
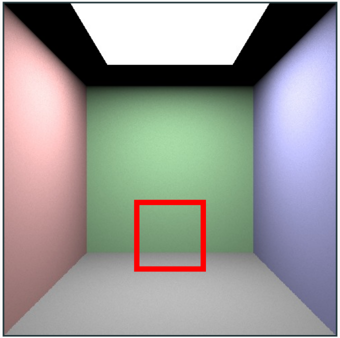


Difference

Line sampling — convergence

Cornell box



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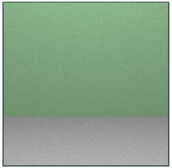
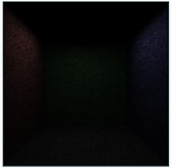
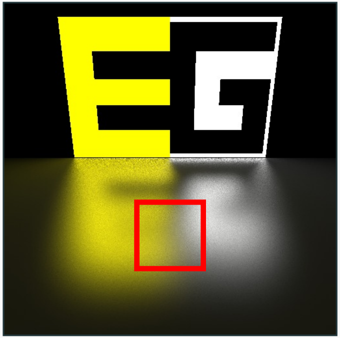




Difference

Eurographics logo



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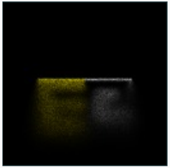

Difference

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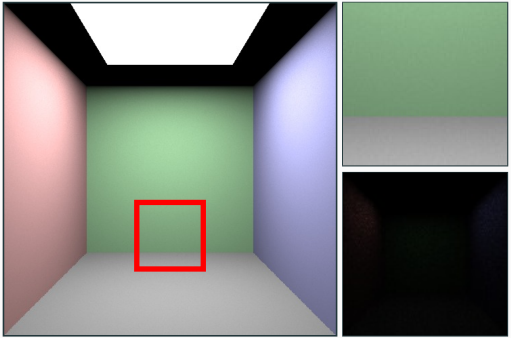
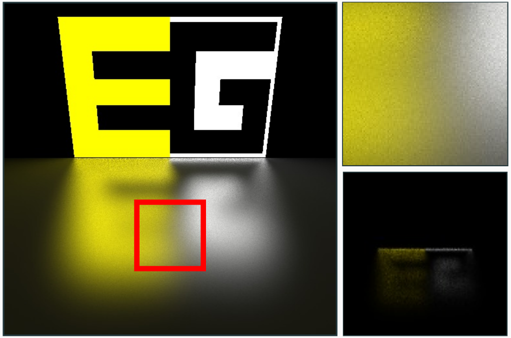
Cornell box Inset



Difference

Eurographics logo Inset



Difference

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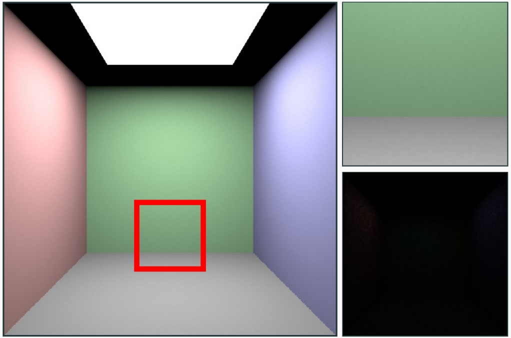
Cornell box

Inset



Difference

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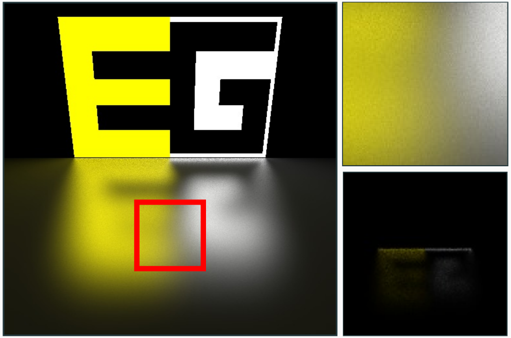
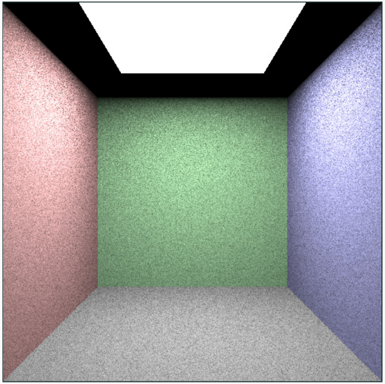
Inset

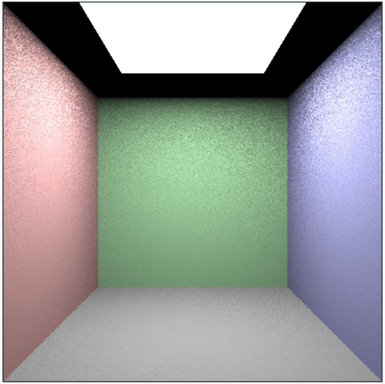


Difference

Importance sampling

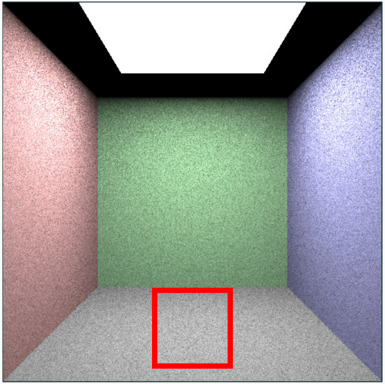


Line sampling

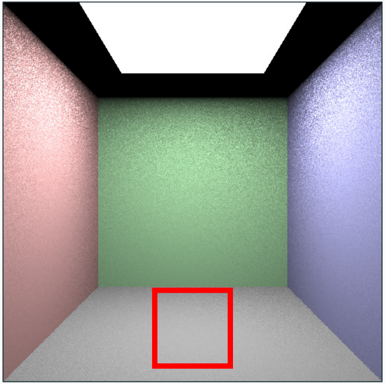


Point sampling

Importance sampling

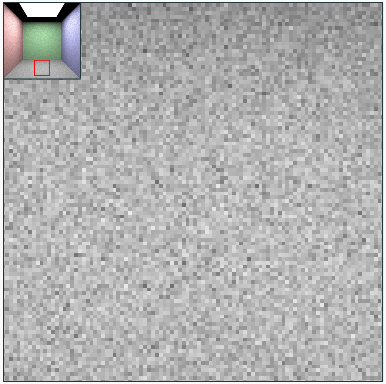


Line sampling

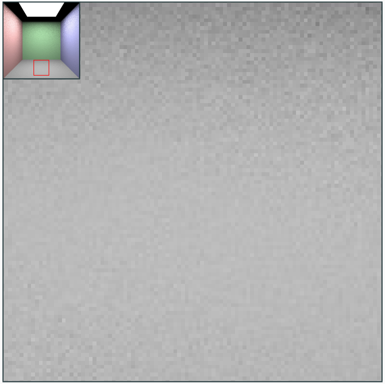


Point sampling

Importance sampling



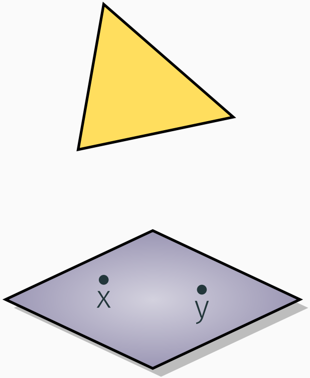
Line sampling



Point sampling

Importance sampling

Line sample contribution is correlated to its **length**



Importance sampling

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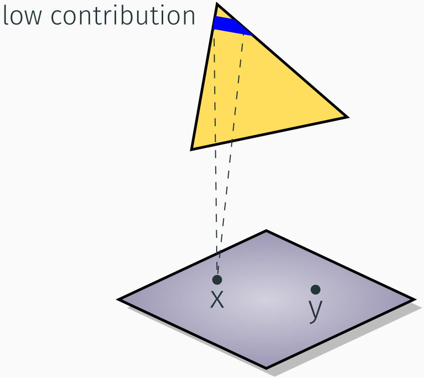

Importance sampling

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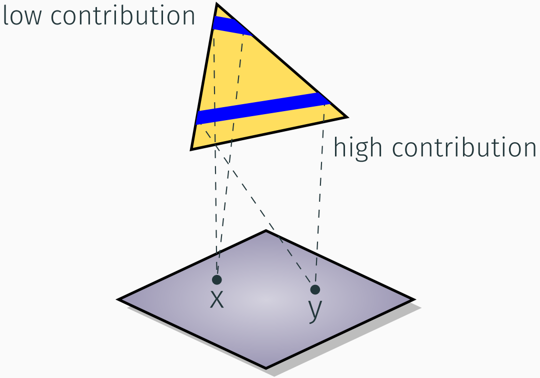
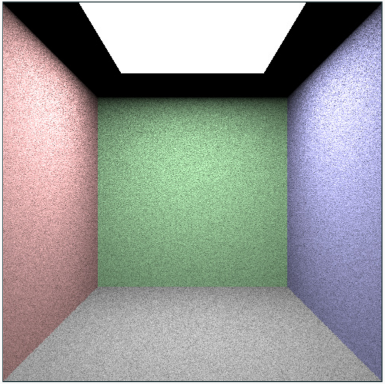
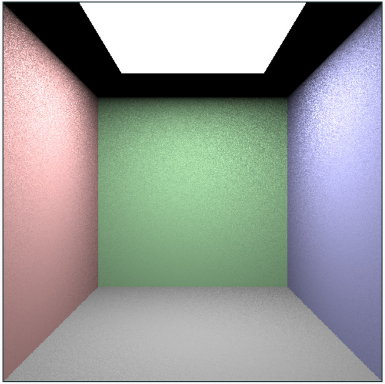


Importance sampling

Solution: importance sampling with the pdf $\simeq l$.



Uniform sampling



Importance sampling

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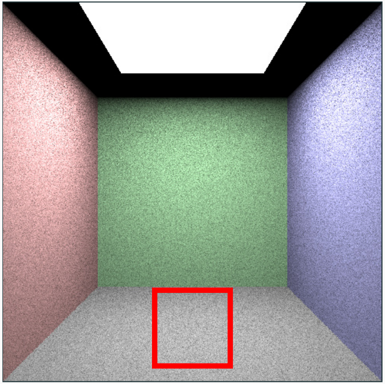
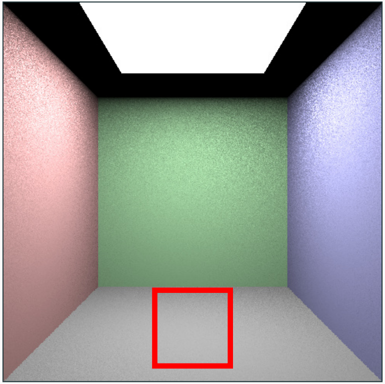


Uniform sampling



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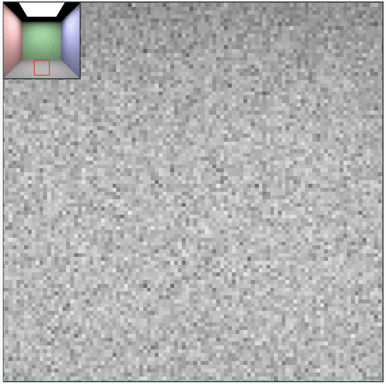
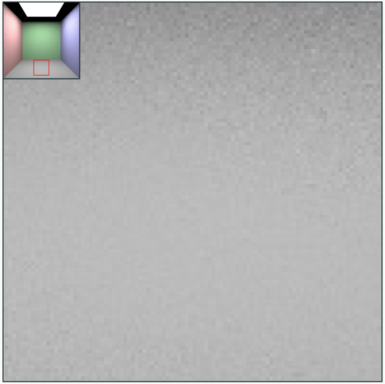
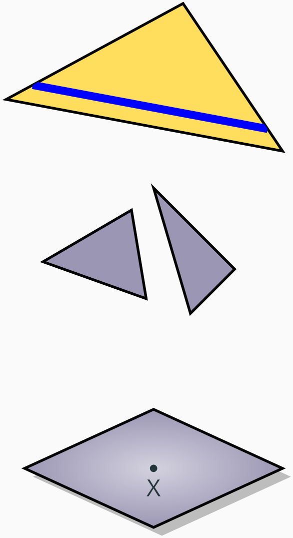


Uniform sampling

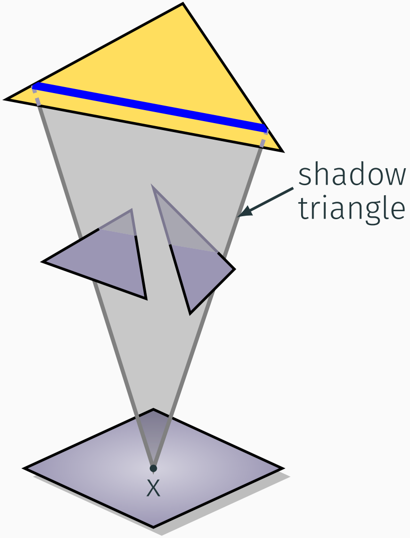


Importance sampling

Visibility evaluation

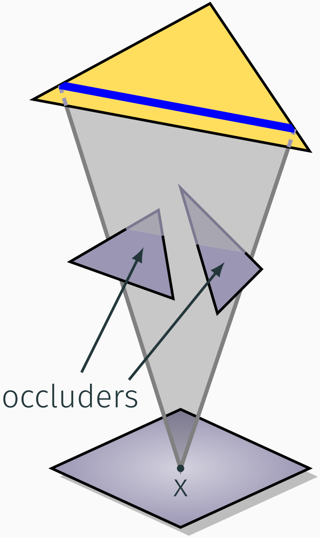


Visibility evaluation



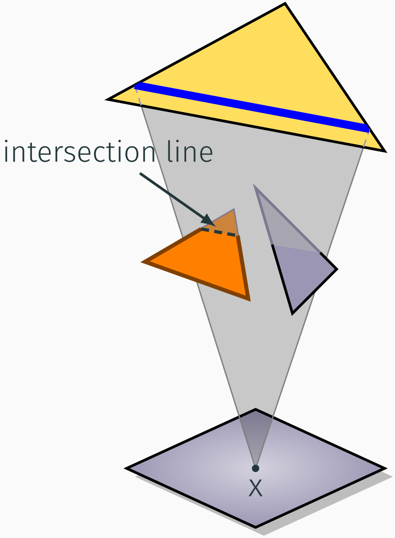
Visibility evaluation

- Use acceleration structure to find *occluders* overlapping the *shadow triangle*.



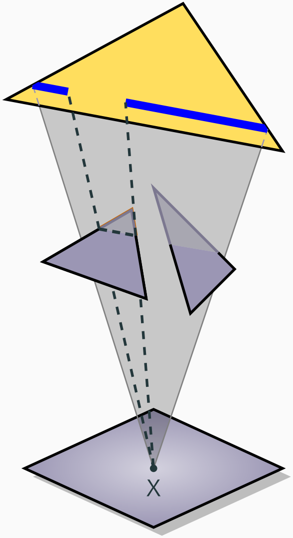
Visibility evaluation

- Use acceleration structure to find *occluders* overlapping the *shadow triangle*.
- Find intersection between occluders and *shadow triangle*.



Visibility evaluation

- Use acceleration structure to find *occluders* overlapping the *shadow triangle*.
- Find intersection between occluders and *shadow triangle*.
- Back projection on line sample.



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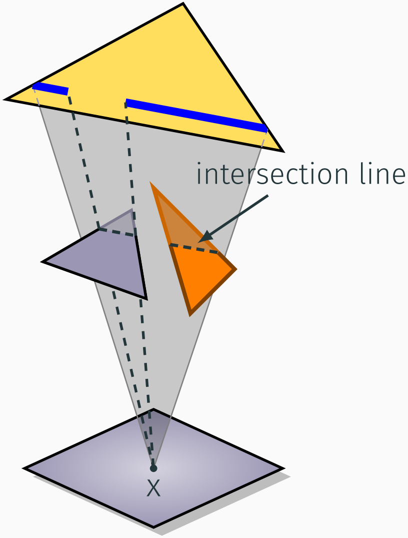

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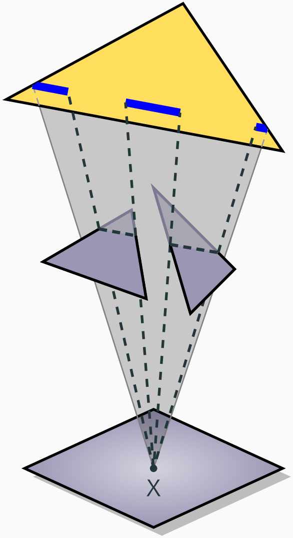

Summary

Summary

What do we have so far:

- generate importance sampled line samples

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- determine the visible parts of the line sample

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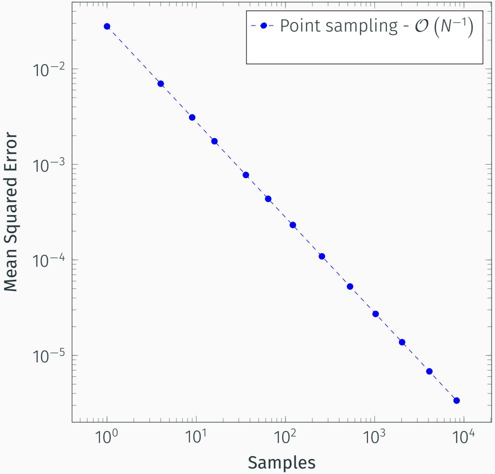
What do we have so far:

- generate importance sampled line samples
- determine the visible parts of the line sample
- evaluate shading of diffuse and Phong materials

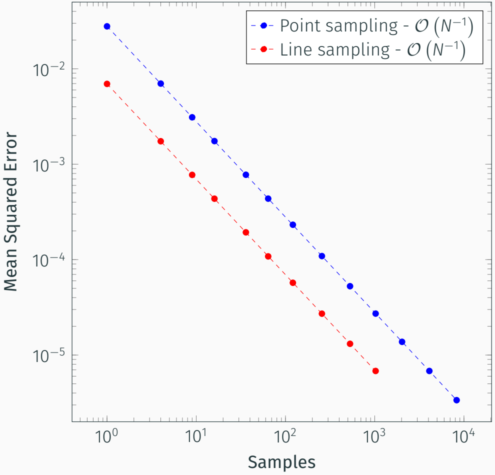
Results

Sponza



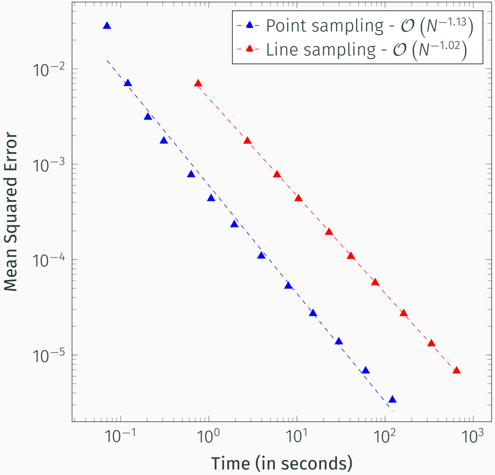
Point sampling - $\mathcal{O}(N^{-1})$
Mean Squared Error
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
Samples

Sponza



Point sampling - $\mathcal{O}(N^{-1})$
Line sampling - $\mathcal{O}(N^{-1})$
10^{-2}
10^{-3}
10^{-4}
10^{-5}
Mean Squared Error
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
Samples

Sponza

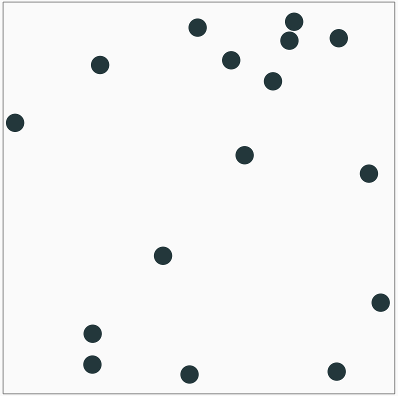


Point sampling - $\mathcal{O}(N^{-1.13})$
Line sampling - $\mathcal{O}(N^{-1.02})$
Mean Squared Error
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Time (in seconds)

Monte Carlo — convergence

- Convergence of Monte Carlo with independent samples:

$$\text{MSE} = \mathcal{O}\left(N^{-1}\right)$$



Random sampling

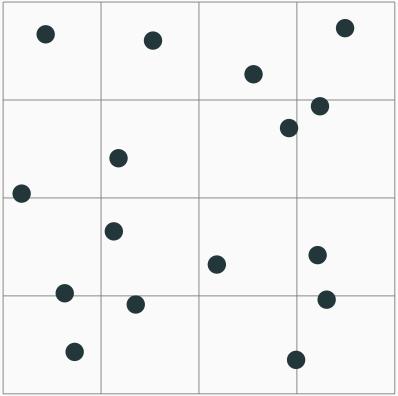
Monte Carlo — convergence

- Convergence of Monte Carlo with independent samples:

$$\mathrm{MSE} = \mathcal{O}\left(N^{-1}\right)$$

- Convergence of Monte Carlo with stratified samples: [Mitchell, 1996]
    - function with bounded first derivative:

$$\mathrm{MSE} = \mathcal{O}\left(N^{-1-2/d}\right)$$



Stratified sampling

Monte Carlo — convergence

- Convergence of Monte Carlo with independent samples:

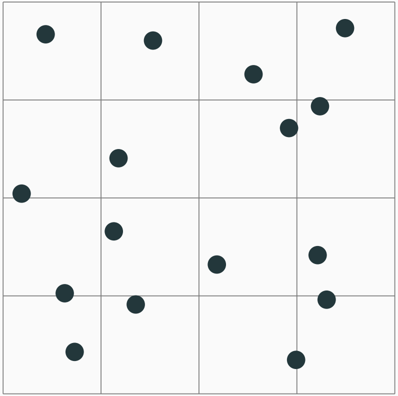
$$\text{MSE} = \mathcal{O}\left(N^{-1}\right)$$

- Convergence of Monte Carlo with stratified samples: [Mitchell, 1996]
  - function with bounded first derivative:

$$\text{MSE} = \mathcal{O}\left(N^{-1-2/d}\right)$$

  - piecewise continuous function:

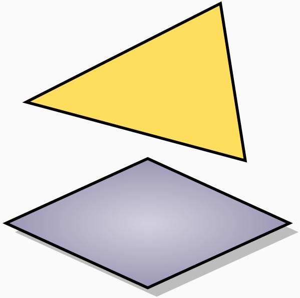
$$\text{MSE} = \mathcal{O}\left(N^{-1-1/d}\right)$$



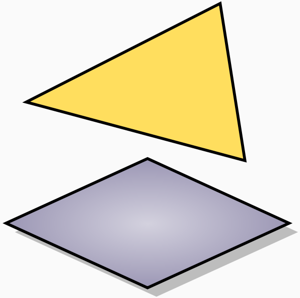
Stratified sampling

Stratified line sampling

$$L_{\text{direct}}(\ldots) = \int_a^b L_{\text{line}}(\ldots)\, du$$



Random sampling



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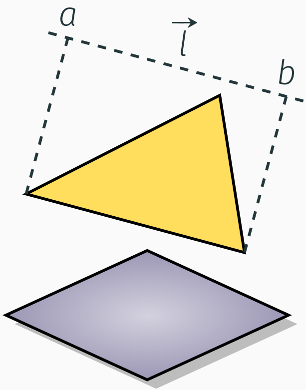
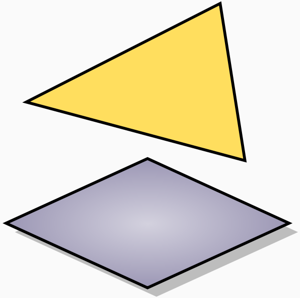


Random sampling



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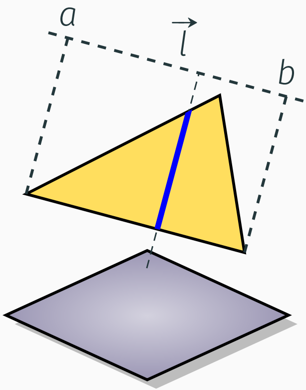
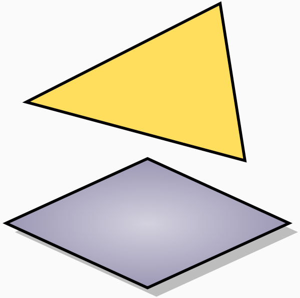


Random sampling



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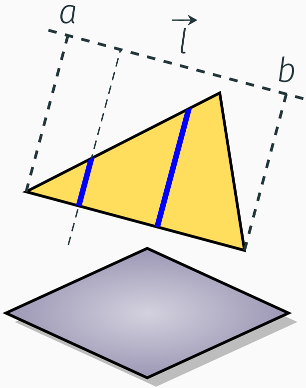
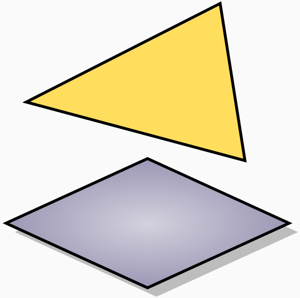


Random sampling



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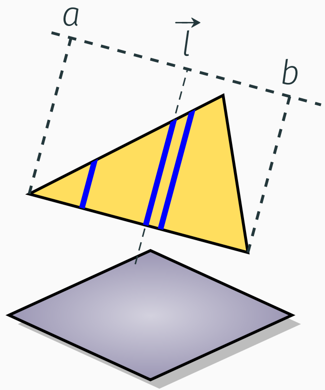
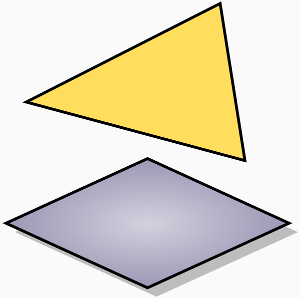


Random sampling



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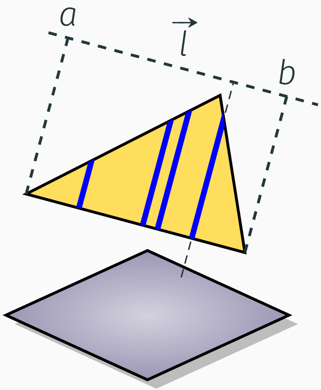
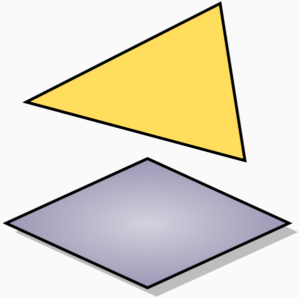


Random sampling



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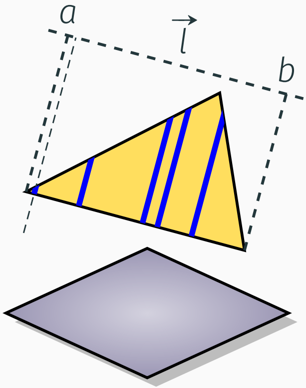
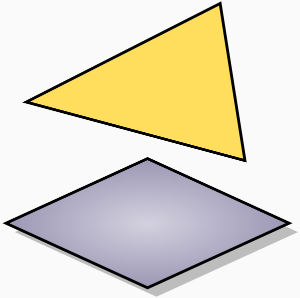


Random sampling



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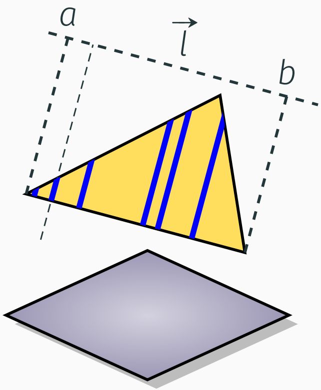
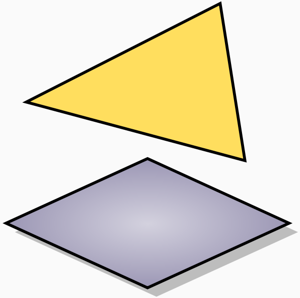


Random sampling



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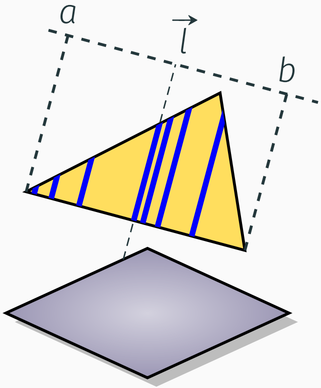
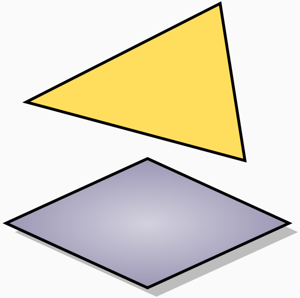


Random sampling



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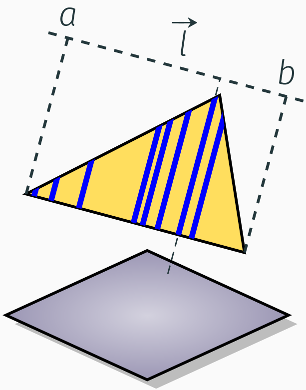
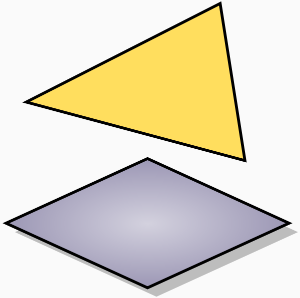


Random sampling



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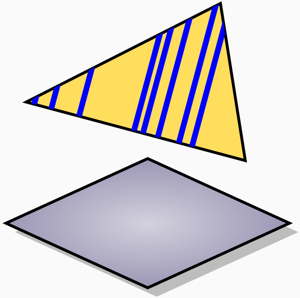

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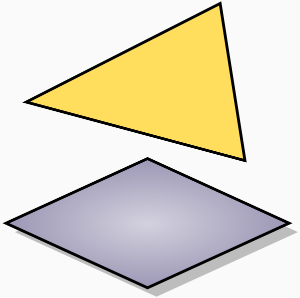

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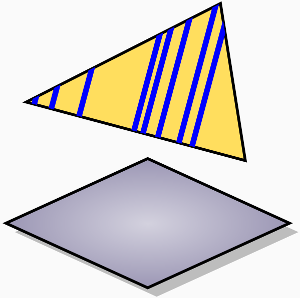
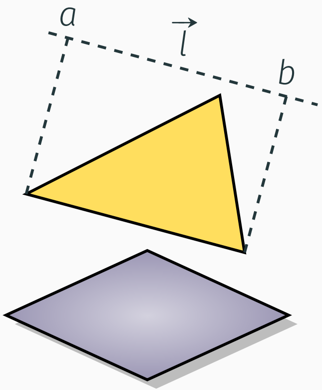


Random sampling



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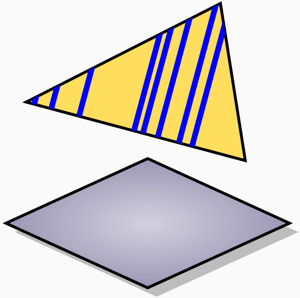

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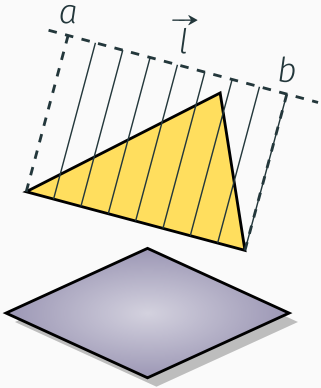

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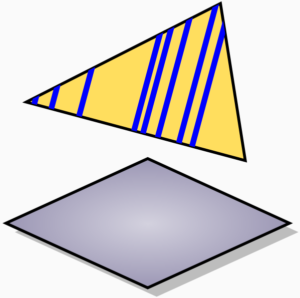

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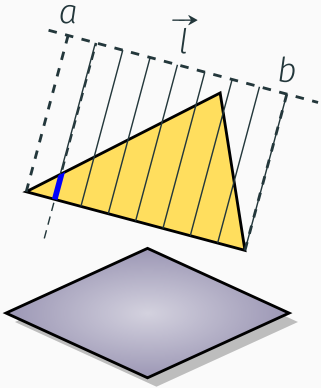
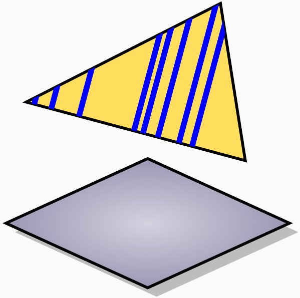


Stratified sampling

Stratified line sampling

$$L_{\text{direct}}\left(\ldots\right) = \int_a^b L_{\text{line}}\left(\ldots\right) du$$



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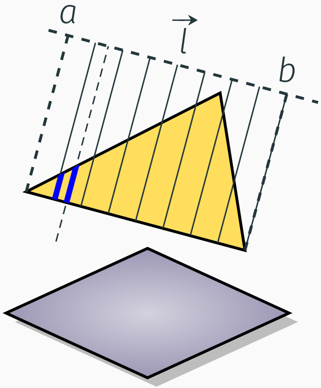

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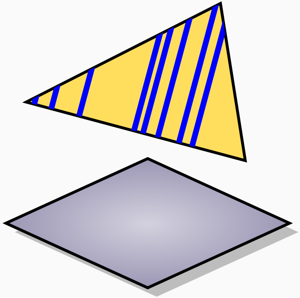

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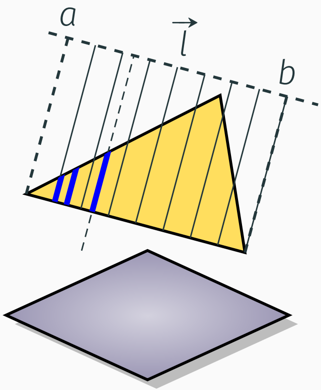

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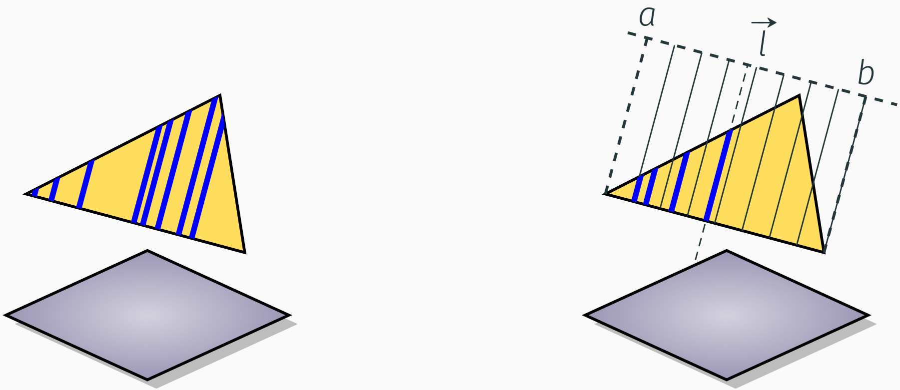

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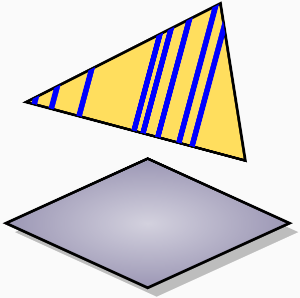
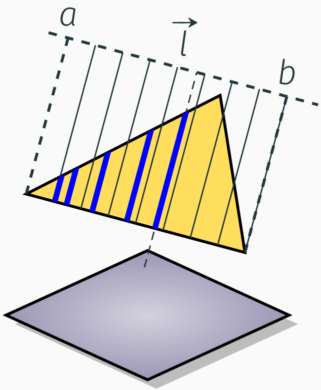


Random sampling



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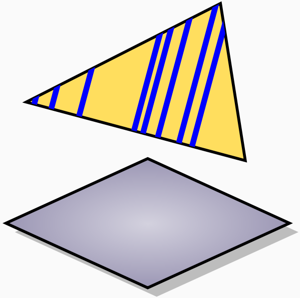

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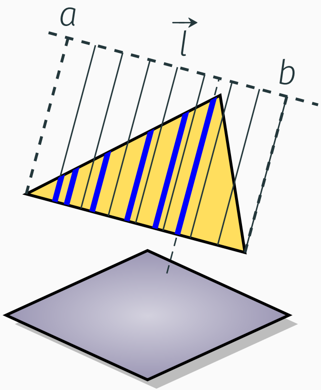

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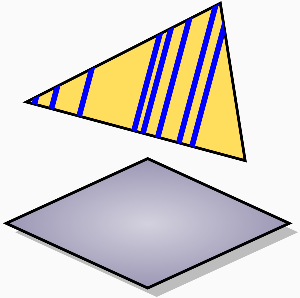

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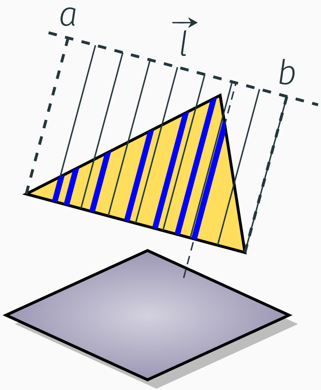

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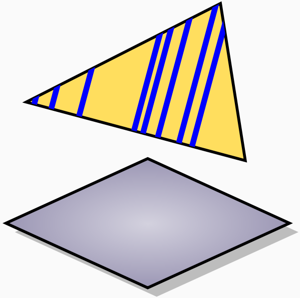
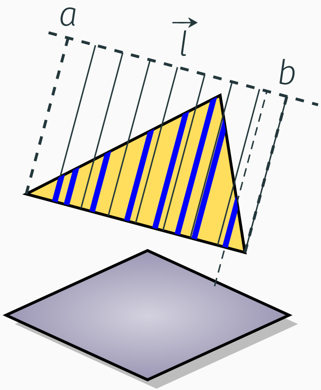


Random sampling



Stratified sampling

Monte Carlo — convergence

Point sampling

Line sampling

Monte Carlo — convergence

Point sampling

- function with bounded first derivative:

$$\text{MSE} = \mathcal{O}\left(N^{-2}\right)$$

Line sampling

- function with bounded first derivative:

$$\text{MSE} = \mathcal{O}\left(N^{-3}\right)$$

Monte Carlo — convergence

Point sampling

- function with bounded first derivative:

$$\text{MSE} = \mathcal{O}\left(N^{-2}\right)$$

- piecewise continuous function:

$$\text{MSE} = \mathcal{O}\left(N^{-1.5}\right)$$

Line sampling

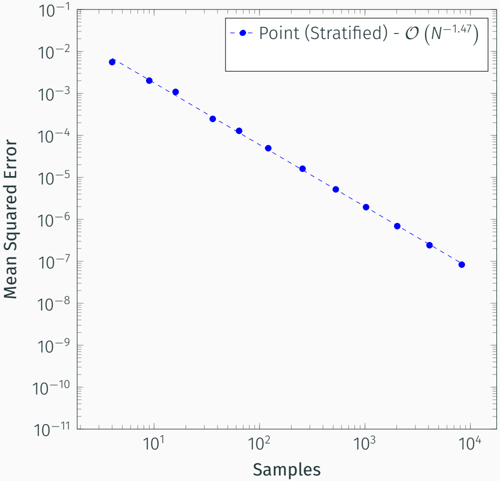
- function with bounded first derivative:

$$\text{MSE} = \mathcal{O}\left(N^{-3}\right)$$

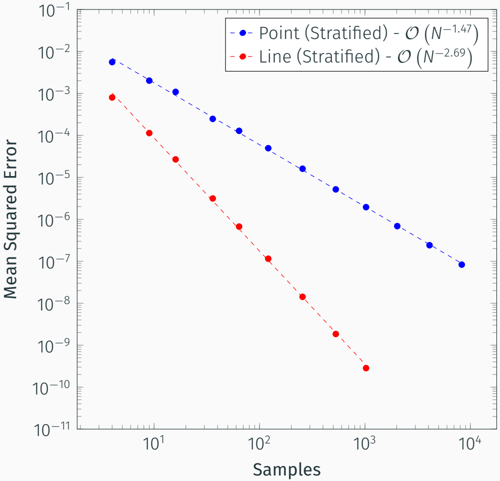
- piecewise continuous function:

$$\text{MSE} = \mathcal{O}\left(N^{-2}\right)$$

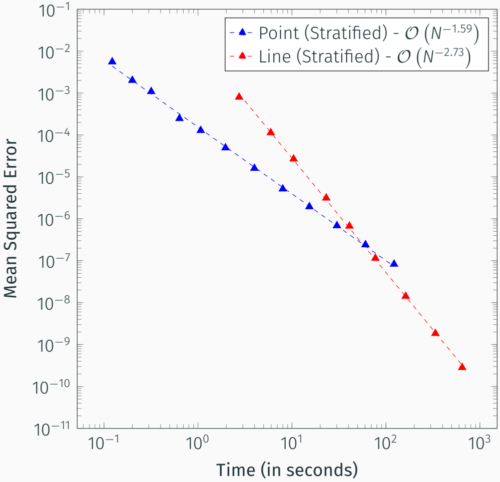
Sponza — stratified sampling



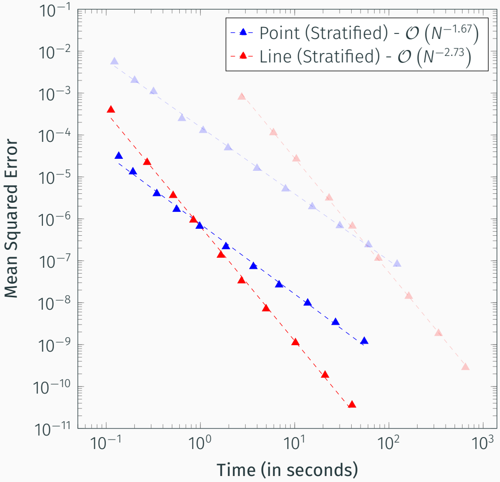
Sponza — stratified sampling



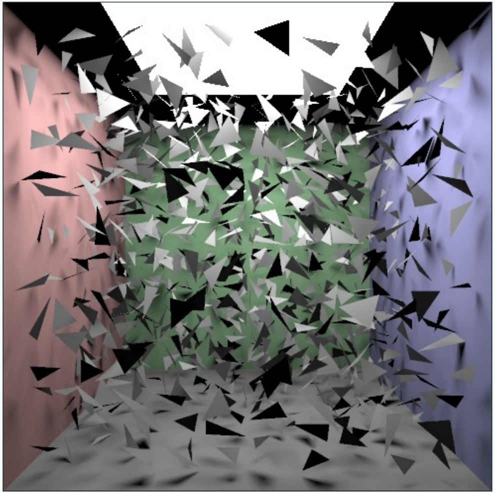
Sponza — stratified sampling

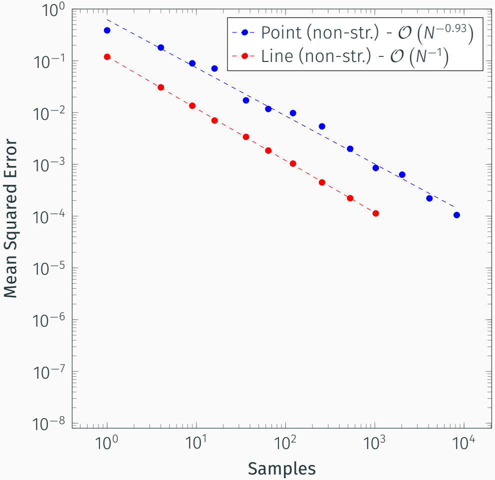


Sponza — small area light

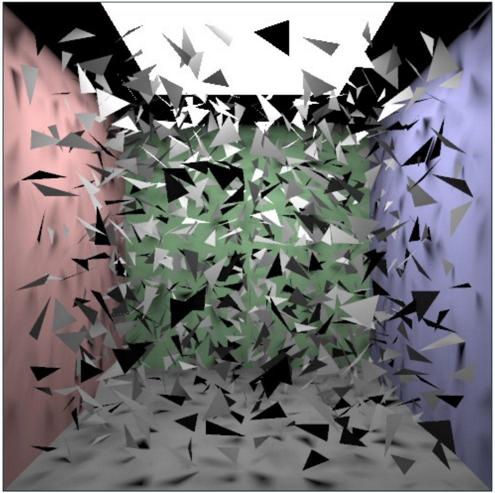


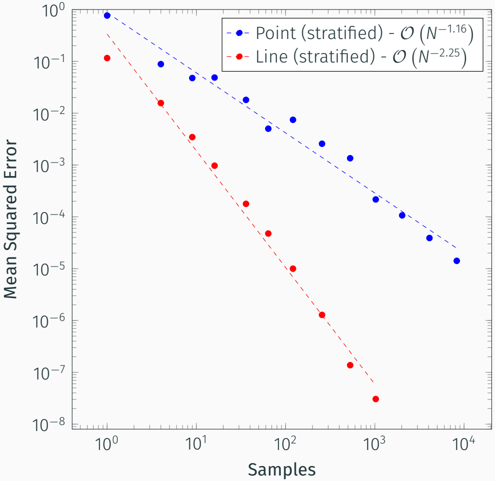
Cornell Box with triangles – non-stratified sampling



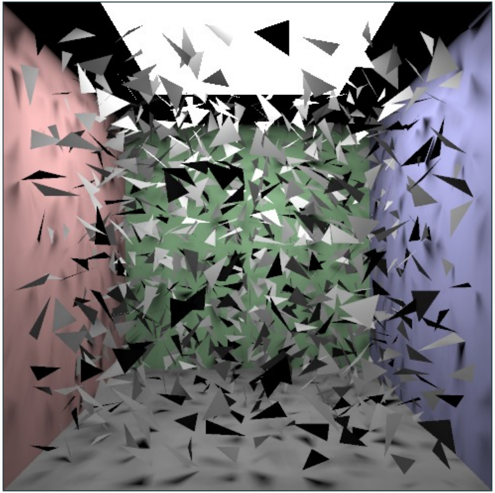


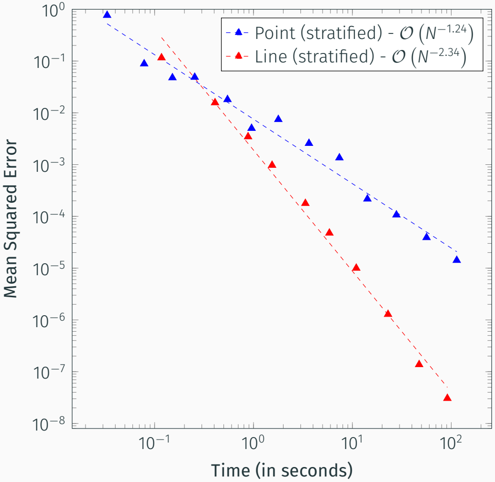
Cornell Box with triangles — stratified sampling



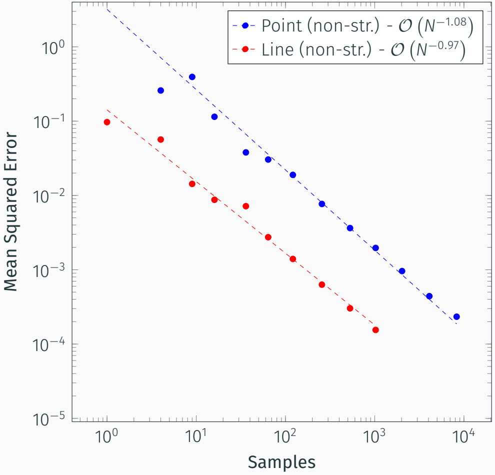


Cornell Box with triangles — stratified sampling

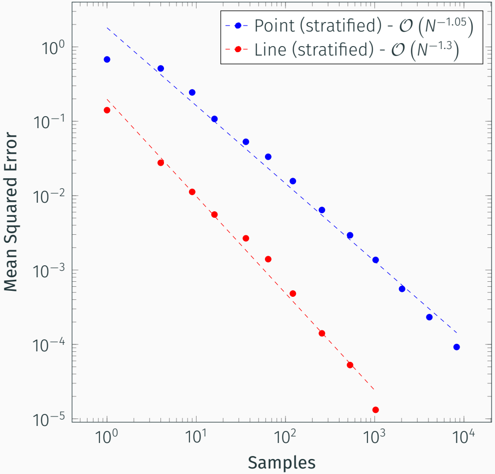




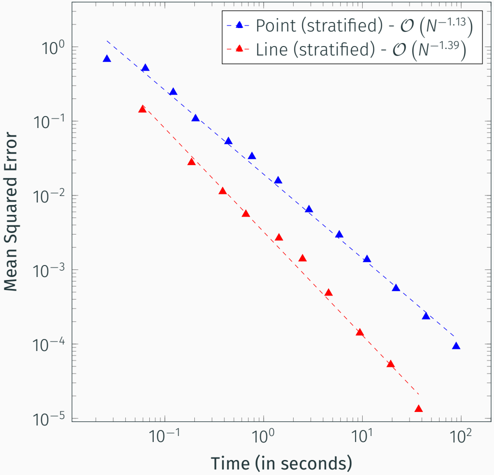
Eurographics Logo – non-stratified sampling



Eurographics Logo — stratified sampling

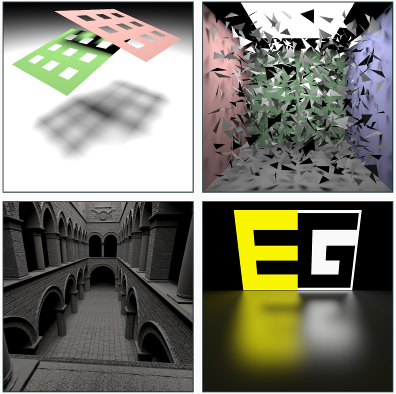


Eurographics Logo — stratified sampling



Conclusion

- Line sampling for direct illumination
  - unbiased images
  - higher order of convergence
- Future work
  - more material models
  - optimize visibility evaluation
  - alternative methods for line sampling



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Poulin, P. and Amanatides, J. (1991).
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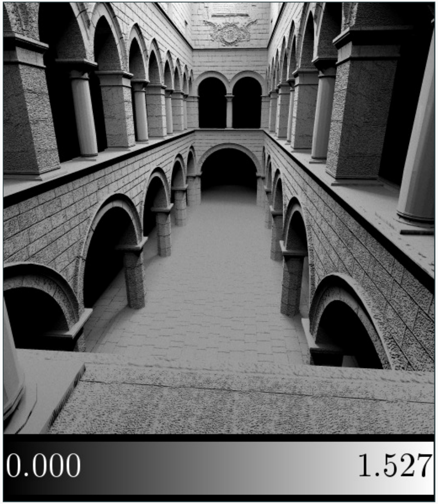
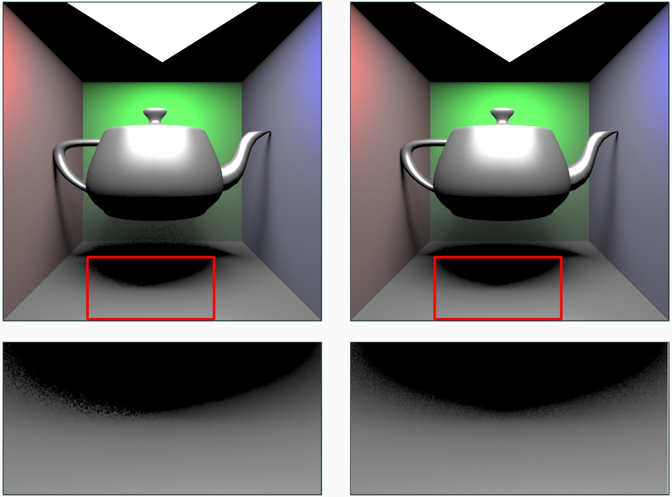
Visibility evaluation



Line sampling — direction

Fixed direction



Stratified direction