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Probabilistic Visibility Evaluation for Direct Illumination

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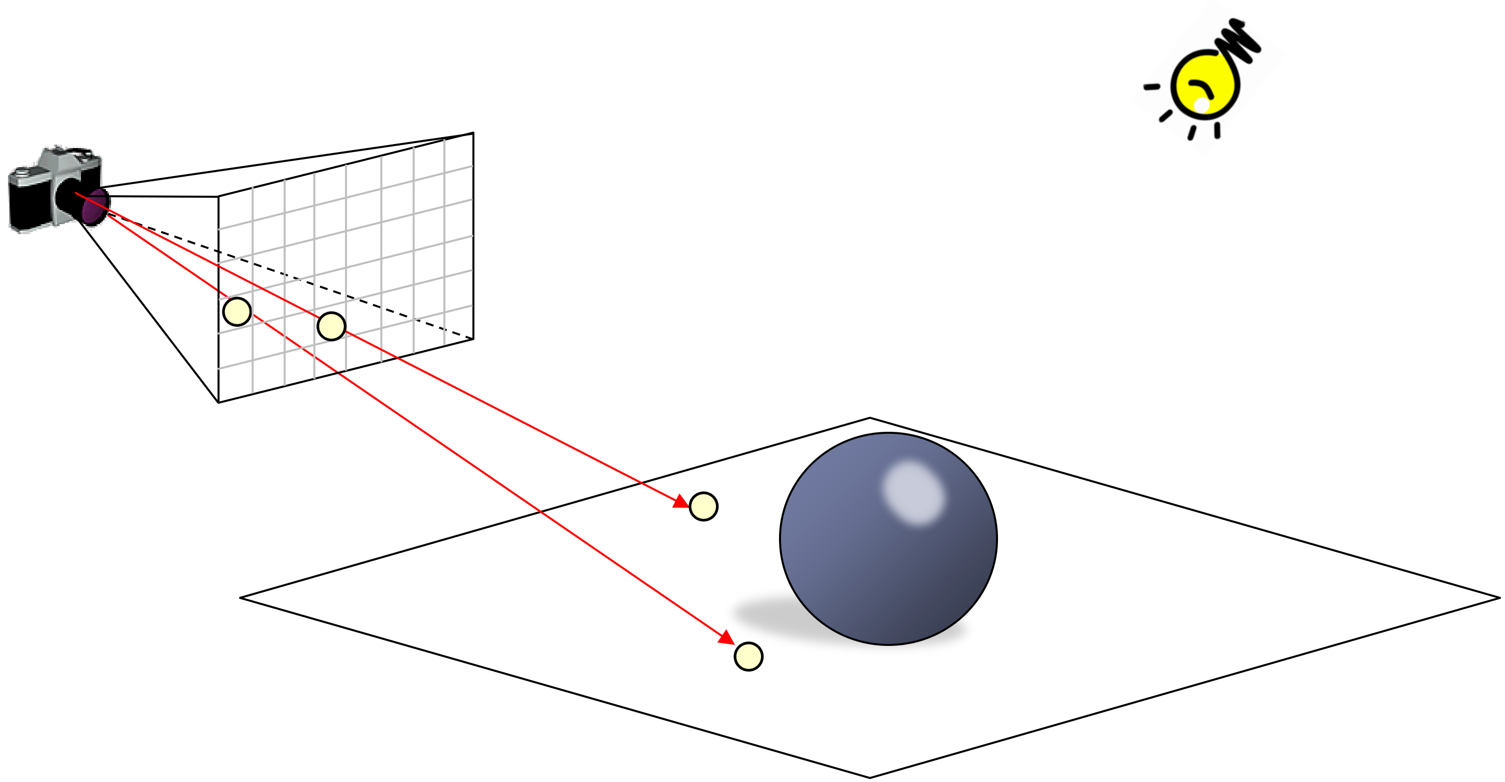
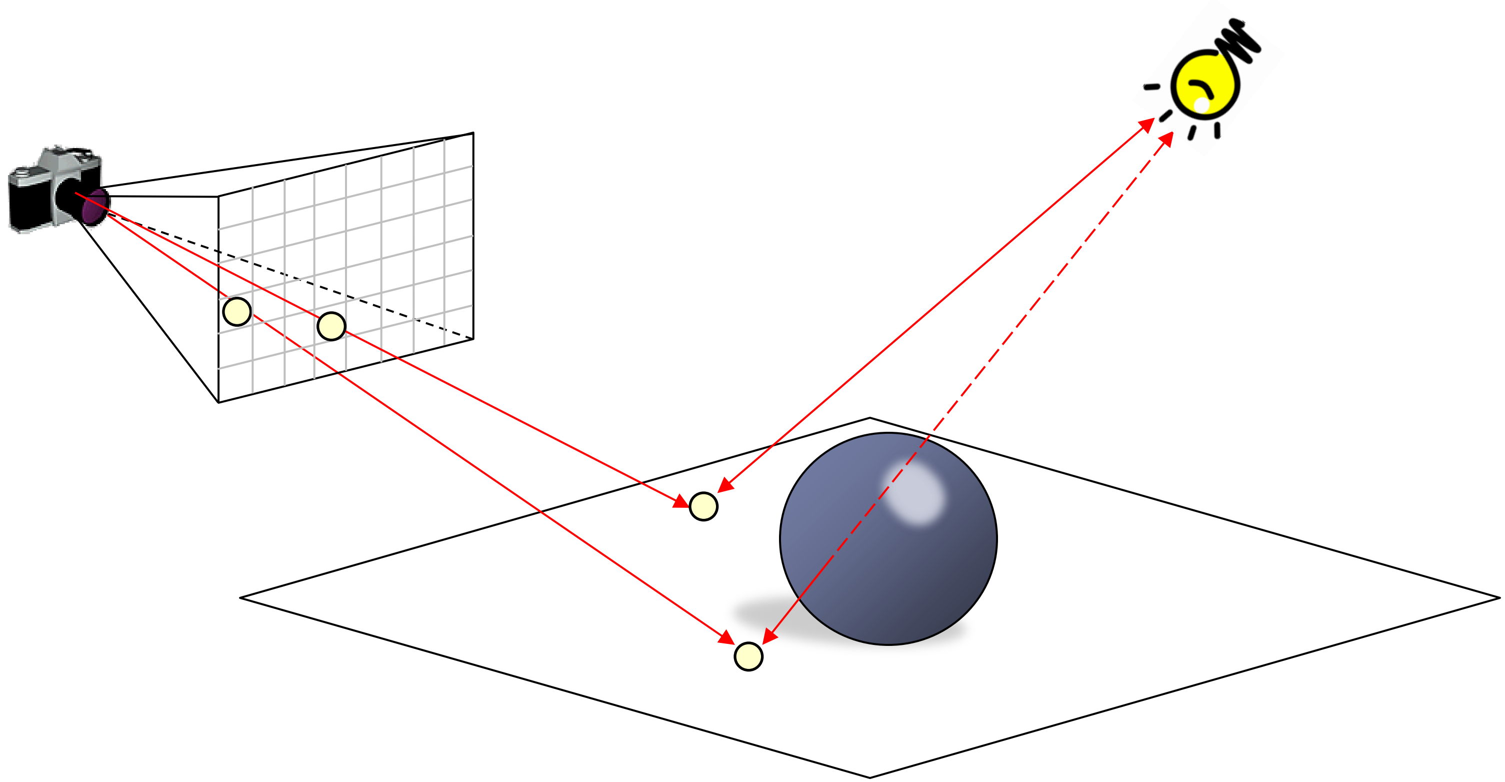
Type of visibility queries in graphics

- "Find the closest intersection point"



Type of visibility queries in graphics

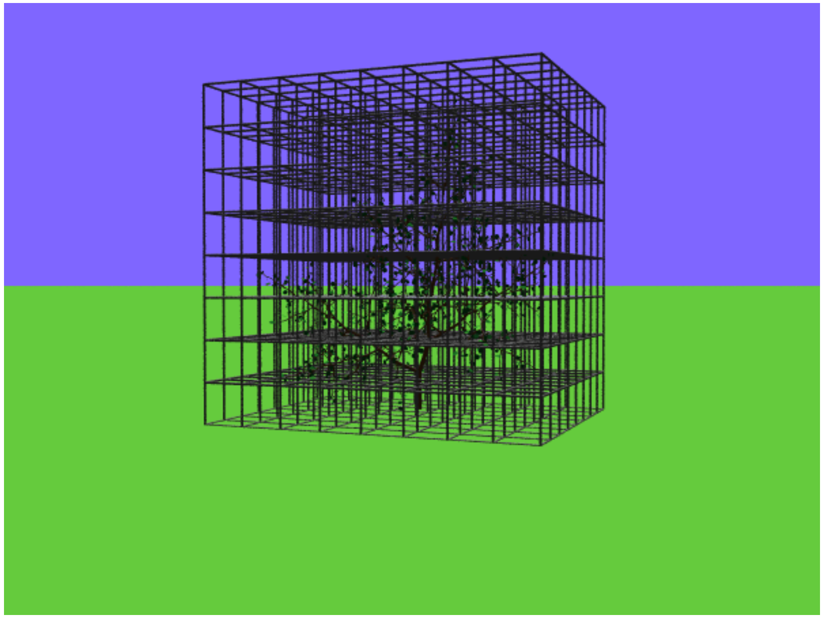
- "Are two points mutually visible?"

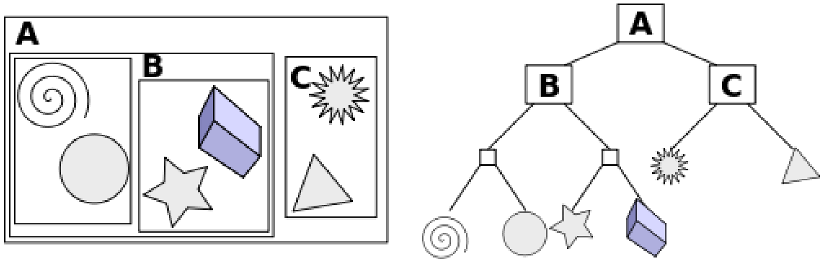


$$L(x) = \int_{Source} f_r(\ldots) \cdot L_e(\ldots) \cdot \frac{\cos(\ldots)\cos(\ldots)}{r_{xy}^2} \cdot V(x,y) \cdot dA_y$$

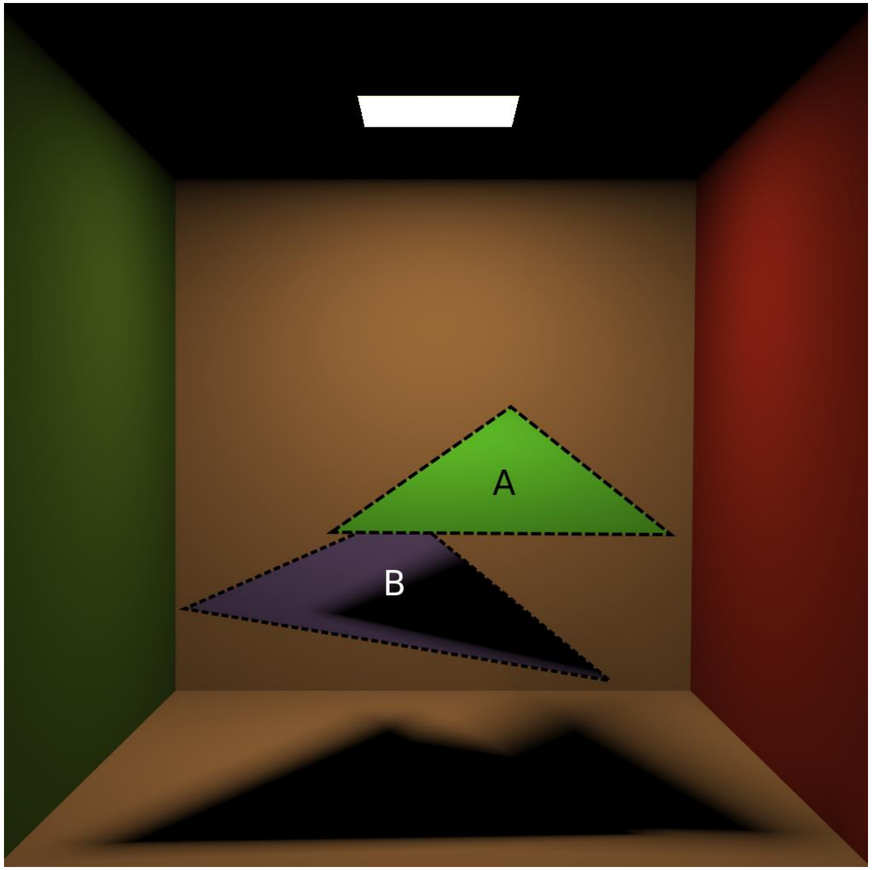
Efficient visibility evaluations

- Spatial grids
- Bounding volume hierarchies
- Light buffer
- Directional structures
- …





Probabilistic Visibility: Theory



$$V(x,y) = V_A(x,y) \cdot V_B(x,y)$$

$$(1-a)(1-b) = 1 - a - b + ab$$

$$ab = a + b + \overline{\bar{a}b} - 1$$

$$V(x,y) = V_A(x,y) + V_B(x,y) + \overline{V_A(x,y)} \cdot \overline{V_B(x,y)} - 1$$

Probabilistic Visibility: Theory

- Monte Carlo evaluation of a sum:

$$S = s_1 + s_2 + \ldots + s_n$$

- Pick term s_i with probability p_i:

$$\tilde{S} = \frac{s_i}{p_i}$$

$$\tilde{S} = \frac{1}{M} \sum \frac{s_i}{p_i}$$

$$E(\tilde{S}) = S$$

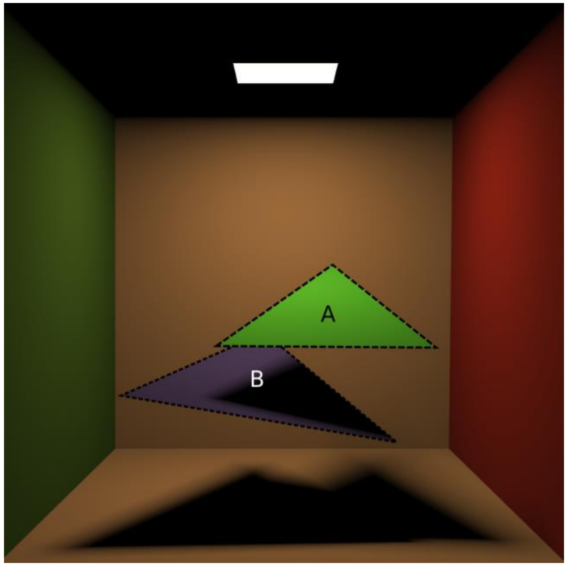
Probabilistic Visibility: Theory

$$V(x,y) = V_A(x,y) + V_B(x,y) + \overline{V_A(x,y)} \cdot \overline{V_B(x,y)} - 1$$

$$\tilde{V}(x,y) = \begin{cases} \dfrac{V_A(x,y)}{p_1} & \text{with probability } p_1 \\ \dfrac{V_B(x,y)}{p_2} & \text{with probability } p_2 \\ \dfrac{\overline{V_A(x,y)} \cdot \overline{V_B(x,y)} - 1}{p_3} & \text{with probability } p_3 \end{cases}$$

Probabilistic Visibility: Example

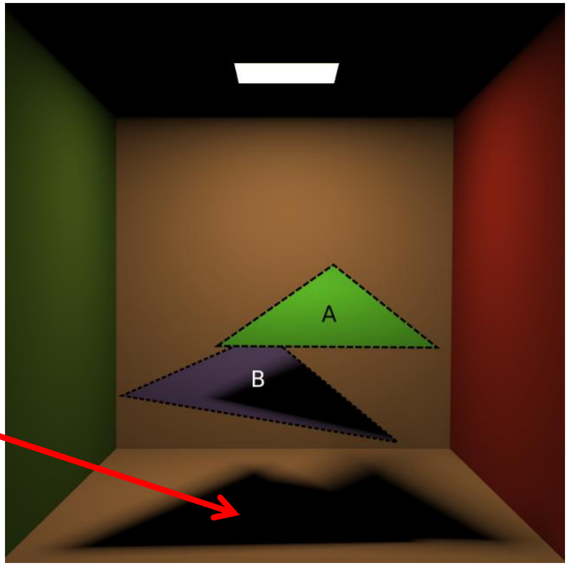
Exact Values			Stochastic Evaluation			Var
V_A	V_B	$V_A.V_B$	$3V_A$	$3V_B$	$3(\bar{V}_A.\bar{V}_B\text{-}1)$	-
0	0	0	0	0	0	0
0	1	0	0	3	-3	6
1	0	0	3	0	-3	6
1	1	1	3	3	-3	8



Probabilistic Visibility: Example

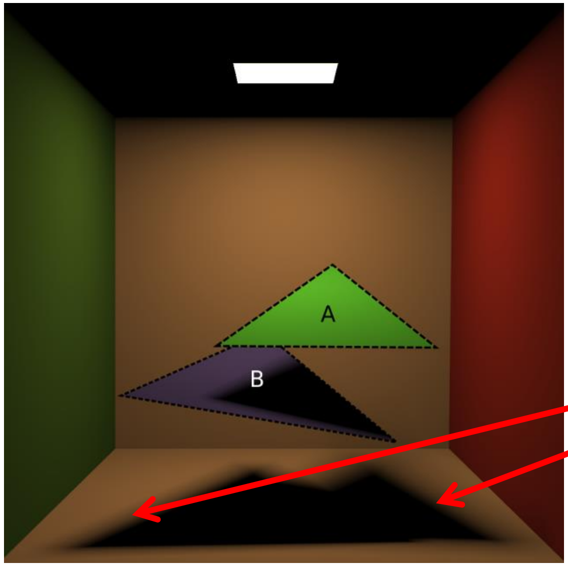
Exact Values			**Stochastic Evaluation**			**Var**
V_A	V_B	$V_A.V_B$	$3V_A$	$3V_B$	$3(\bar{V}_A.\bar{V}_B\text{-}1)$	-
0	0	0	0	0	0	0
0	1	0	0	3	-3	6
1	0	0	3	0	-3	6
1	1	1	3	3	-3	8

both A and B cast shadow



Probabilistic Visibility: Example

Exact Values			Stochastic Evaluation			Var
V_A	V_B	$V_A.V_B$	$3V_A$	$3V_B$	$3(\bar{V}_A.\bar{V}_B\text{-}1)$	-
0	0	0	0	0	0	0
0	1	0	0	3	-3	6
1	0	0	3	0	-3	6
1	1	1	3	3	-3	8

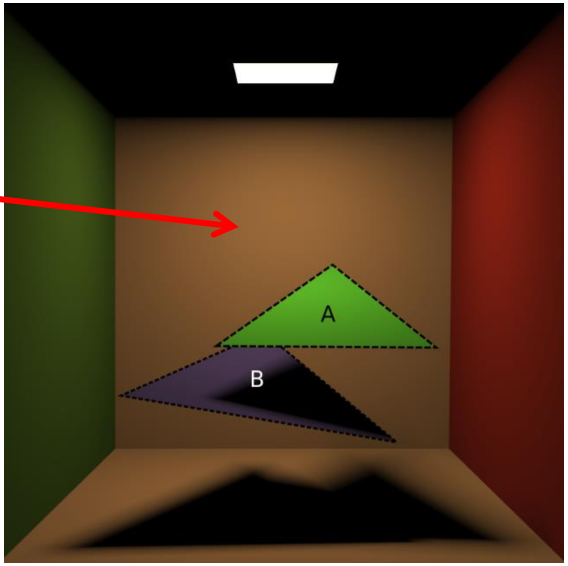


either A or B cast shadow

Probabilistic Visibility: Example

Exact Values			Stochastic Evaluation			Var
V_A	V_B	$V_A.V_B$	$3V_A$	$3V_B$	$3(\bar{V}_A.\bar{V}_B\text{-}1)$	-
0	0	0	0	0	0	0
0	1	0	0	3	-3	6
1	0	0	3	0	-3	6
1	1	1	3	3	-3	8

neither A nor B cast shadow



Probabilistic Visibility: Direct illumination

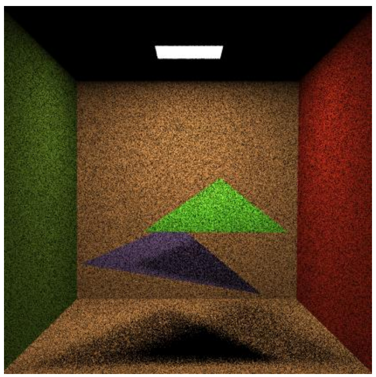
$$L(x) = \int_{Source} f_r(\ldots) \cdot L_e(\ldots) \cdot G(\ldots) \cdot V(x,y) \cdot dA_y$$

$$= \int_{Source} f_r(\ldots) \cdot L_e(\ldots) \cdot G(\ldots) \cdot V_A(x,y) \cdot dA_y$$

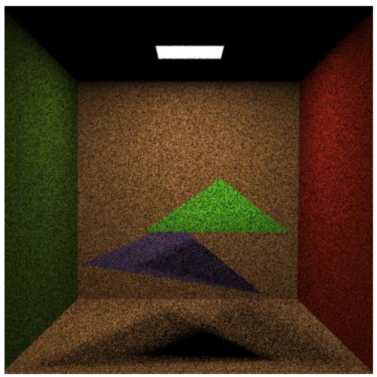
$$+ \int_{Source} f_r(\ldots) \cdot L_e(\ldots) \cdot G(\ldots) \cdot V_B(x,y) \cdot dA_y$$

$$+ \int_{Source} f_r(\ldots) \cdot L_e(\ldots) \cdot G(\ldots) \cdot (\overline{V_A(x,y)}\,\overline{V_B(x,y)} - 1) \cdot dA_y$$

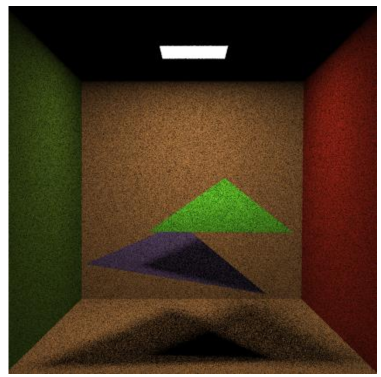
Probabilistic Visibility: Results



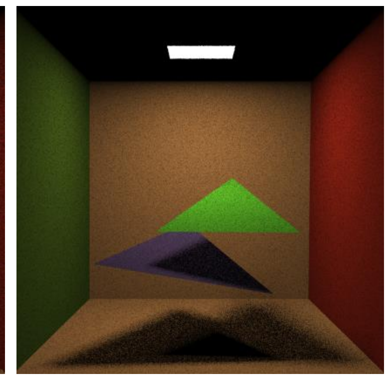
1 shadow ray



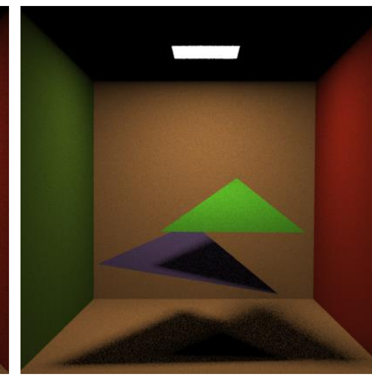
4 shadow ray



16 shadow ray



64 shadow ray



256 shadow ray

Alternative Decomposition

Exact Values			Stochastic Evaluation			Var
V_A	V_B	$V_A.V_B$	$3V_A$	$3V_B$	$3(\bar{V}_A.\bar{V}_B\text{-}1)$	-
0	0	0	0	0	0	0
0	1	0	0	3	-3	6
1	0	0	3	0	-3	6
1	1	1	3	3	-3	8

Exact Values			Stochastic Evaluation			Var
V_A	V_B	$V_A.V_B$	$3V_A\text{-}1$	$3V_B\text{-}1$	$3\bar{V}_A.\bar{V}_B\text{-}1$	-
0	0	0	-1	-1	2	2
0	1	0	-1	2	-1	2
1	0	0	2	-1	-1	2
1	1	1	2	2	-1	2

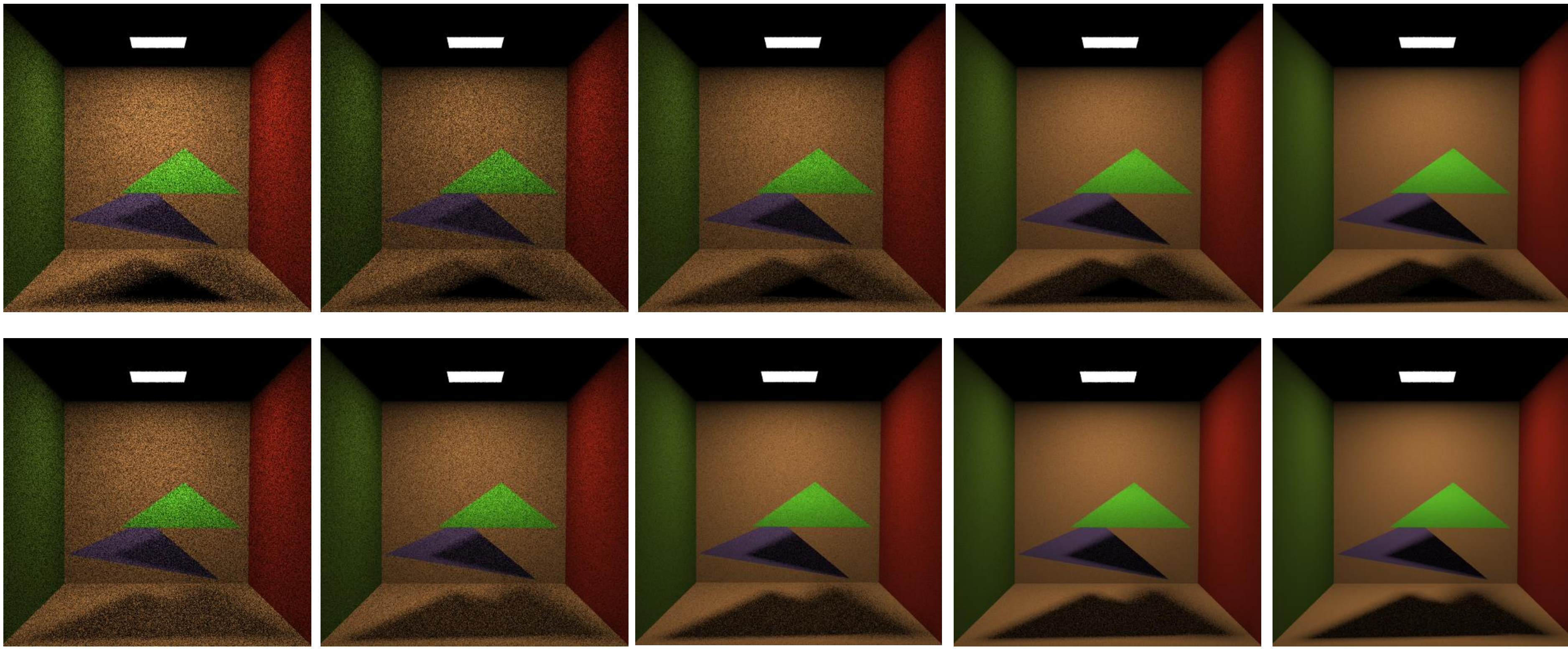
Alternative Decomposition

Exact Values			Stochastic Evaluation			Var
V_A	V_B	$V_A.V_B$	$3V_A$	$3V_B$	$3(\bar{V}_A.\bar{V}_B\text{-}1)$	-
0	0	0	0	0	0	0
0	1	0	0	3	-3	6
1	0	0	3	0	-3	6
1	1	1	3	3	-3	8

Exact Values			Stochastic Evaluation			Var
V_A	V_B	$V_A.V_B$	$3V_A\text{-}1$	$3V_B\text{-}1$	$3\bar{V}_A.\bar{V}_B\text{-}1$	-
0	0	0	-1	-1	2	2
0	1	0	-1	2	-1	2
1	0	0	2	-1	-1	2
1	1	1	2	2	-1	2

Alternative Decomposition

1 shadow ray | 4 shadow rays | 16 shadow rays | 64 shadow rays | 256 shadow rays



Alternative Decomposition

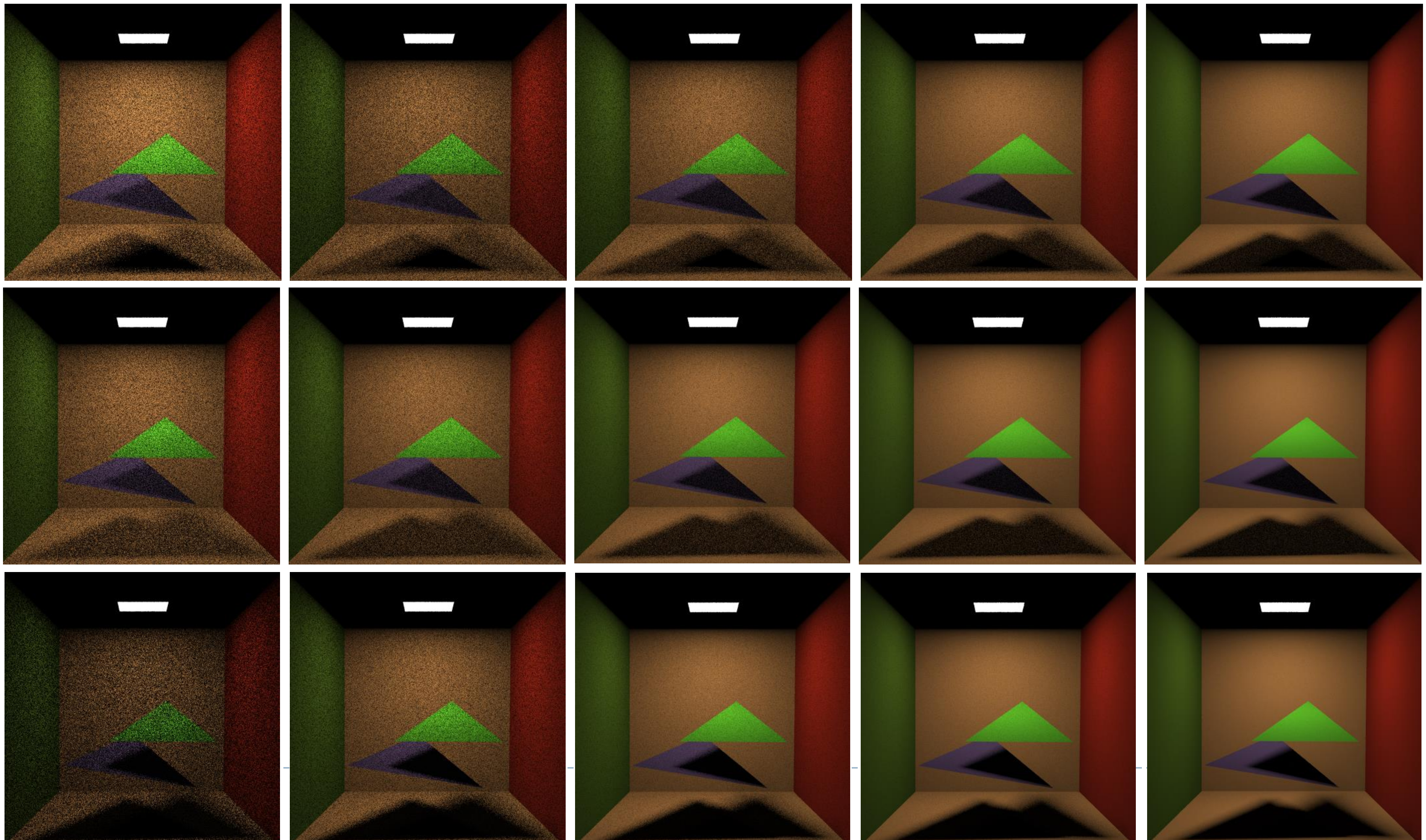
$$(V_A(x,y)+V_B(x,y))^n = V_A(x,y)+V_B(x,y)+(2^n-2)V_A(x,y)V_B(x,y)$$

$$V_A(x,y)V_B(x,y) = -\frac{V_A(x,y)}{(2^n-2)} - \frac{V_B(x,y)}{(2^n-2)} + \frac{(V_A(x,y)+V_B(x,y))^n}{(2^n-2)}$$

Exact Values			**Stochastic Evaluation**			**Var**
V_A	V_B	$V_A.V_B$	$-3V_A/254$	$-3V_B/254$	$3(V_A+V_B)^8/254$	-
0	0	0	0	0	0	0
0	1	0	0	-3/254	3/254	4.65e-5
1	0	0	-3/254	0	3/254	4.65e-5
1	1	1	-3/254	-3/254	768/254	2.048

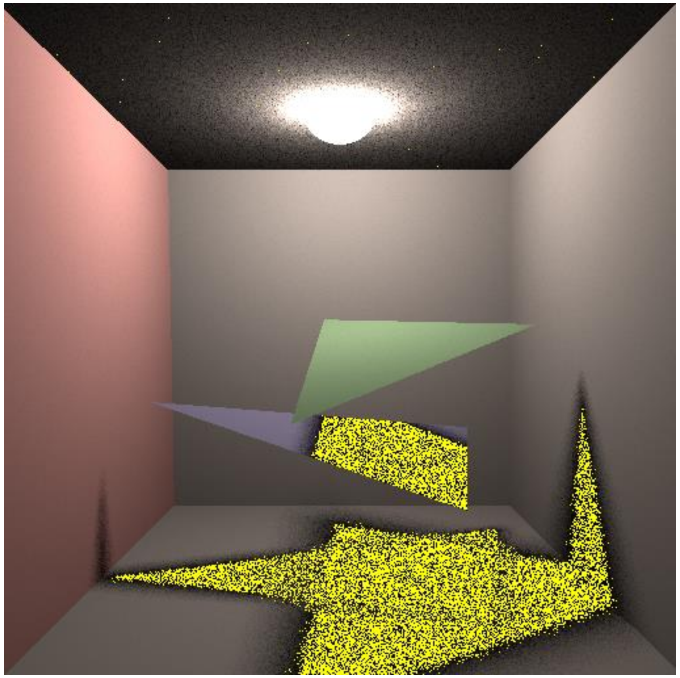
Alternative Decomposition

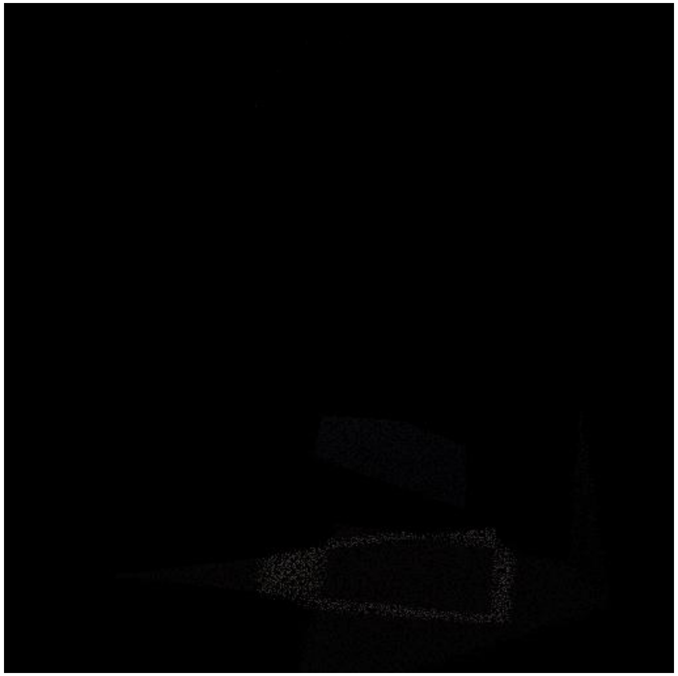
1 shadow ray	4 shadow rays	16 shadow rays	64 shadow rays	256 shadow rays



Discussion

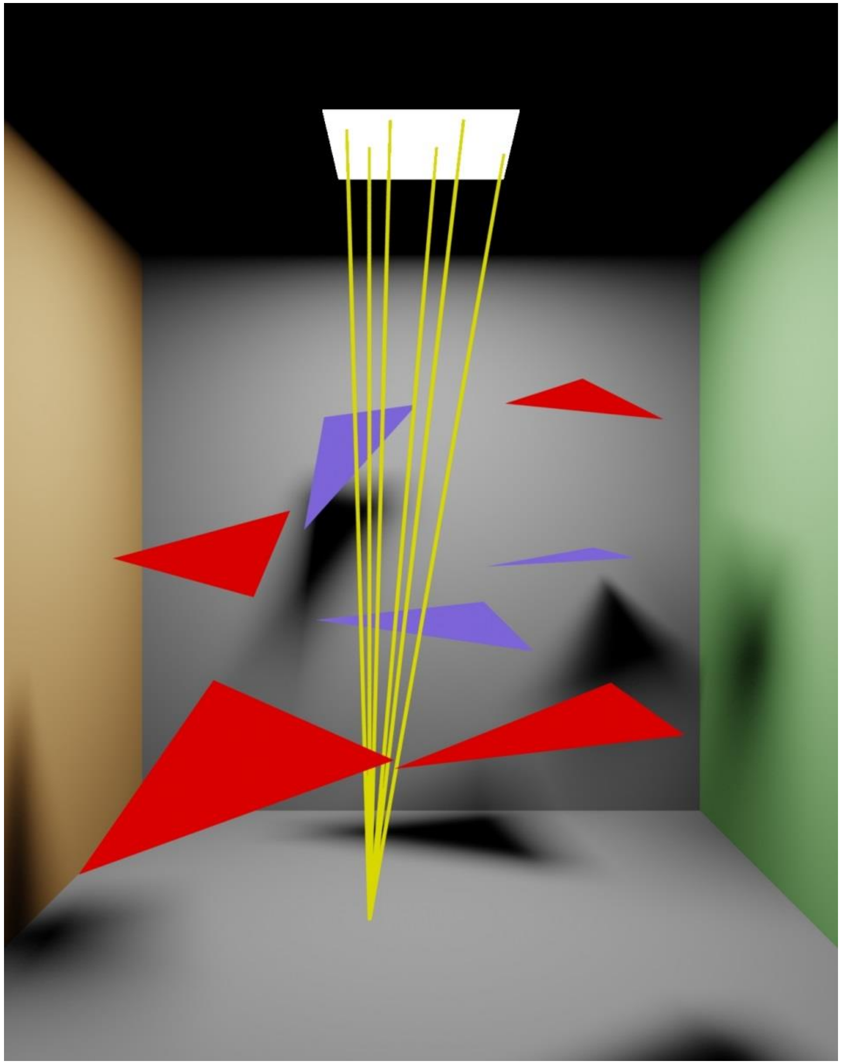
- Negative stochastic visibility evaluation
  - Results in "negative pixels"
  - Clamp "negative" pixels to zero
    ➔ Image too bright on average





Discussion

- Multiple blockers
  - Split in 2 groups A and B (theory still holds)
  - Recursive subdivision
    - Sharp increase in variance
    - Intuition:
      test only 1 polygon for intersection
  - Acceleration grid
    - Evaluate visibility in each cell (negative, zero, positive)
    - Multiply visibility values for each cell
      ➔ sharp increase in variance

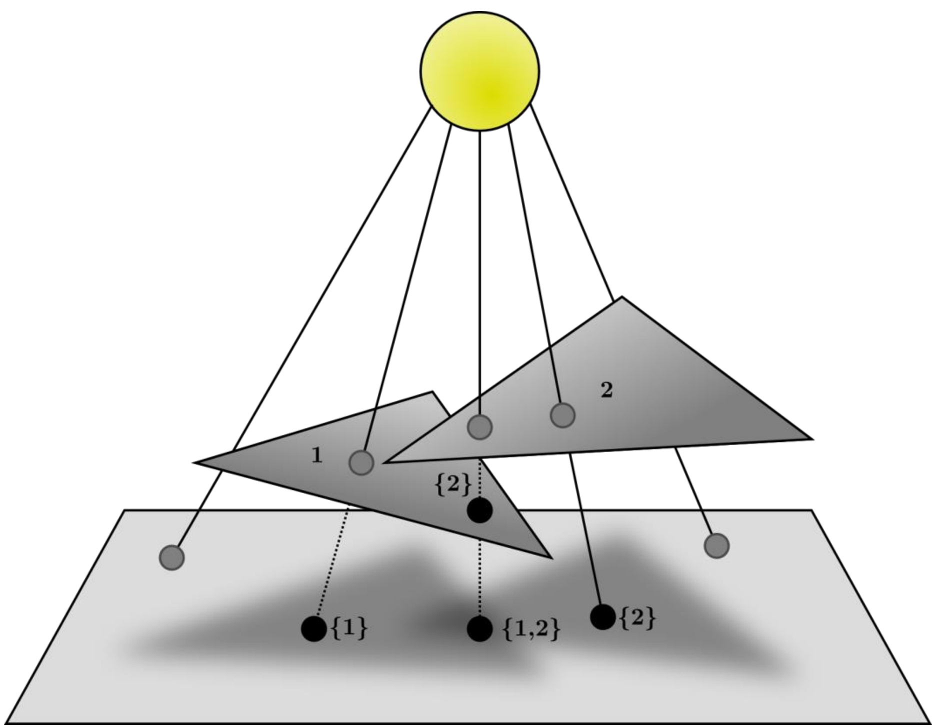


Practical Algorithm

- What would we like?
  - Select potential blockers along entire length of shadow ray (no spatial grid of successive cells)
    ➔ suggests a directional acceleration structure
    - Light buffer
    - Shafts
    - Occlusion Map
- Evaluation:
  - Split set of potential blockers in 2 groups
  - Maintain roughly equal intersection probabilities values for both groups

Phase 1: Construct Occlusion Map

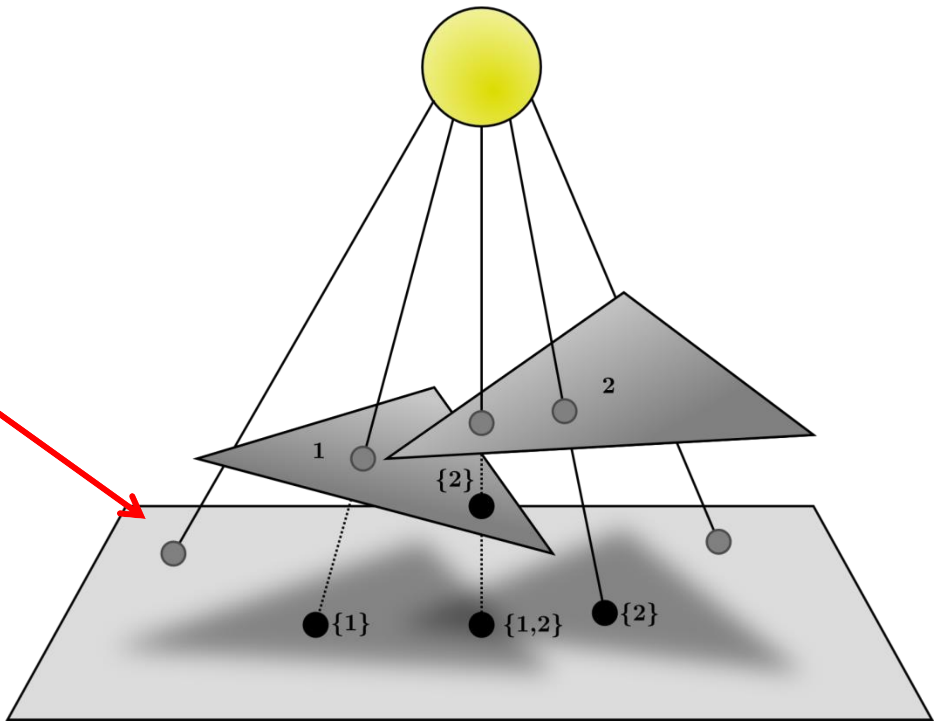
- Construct occlusion map and light map
  - Store locations into kd-tree for fast look-up.



Phase 1: Construct Occlusion Map

- Construct occlusion map and light map
  - Store locations into kd-tree for fast look-up.

light photon



Phase 1: Construct Occlusion Map

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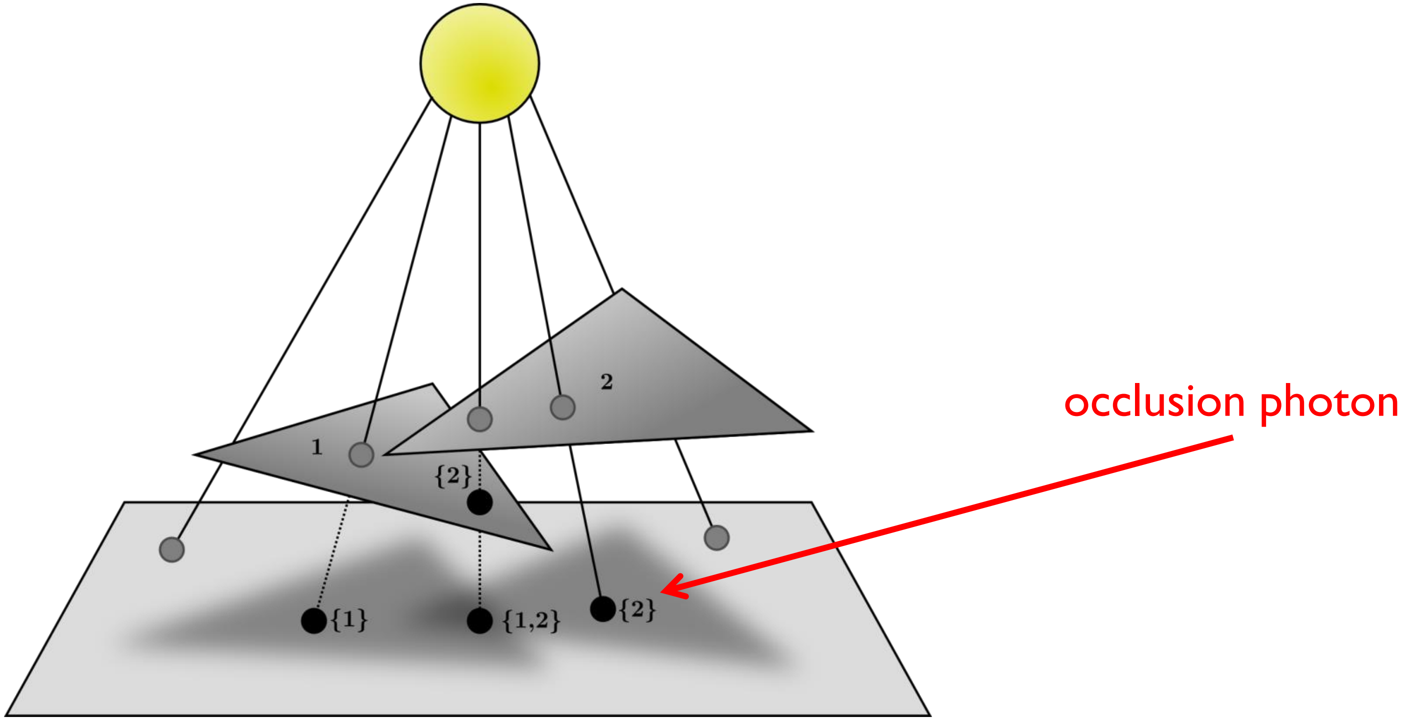
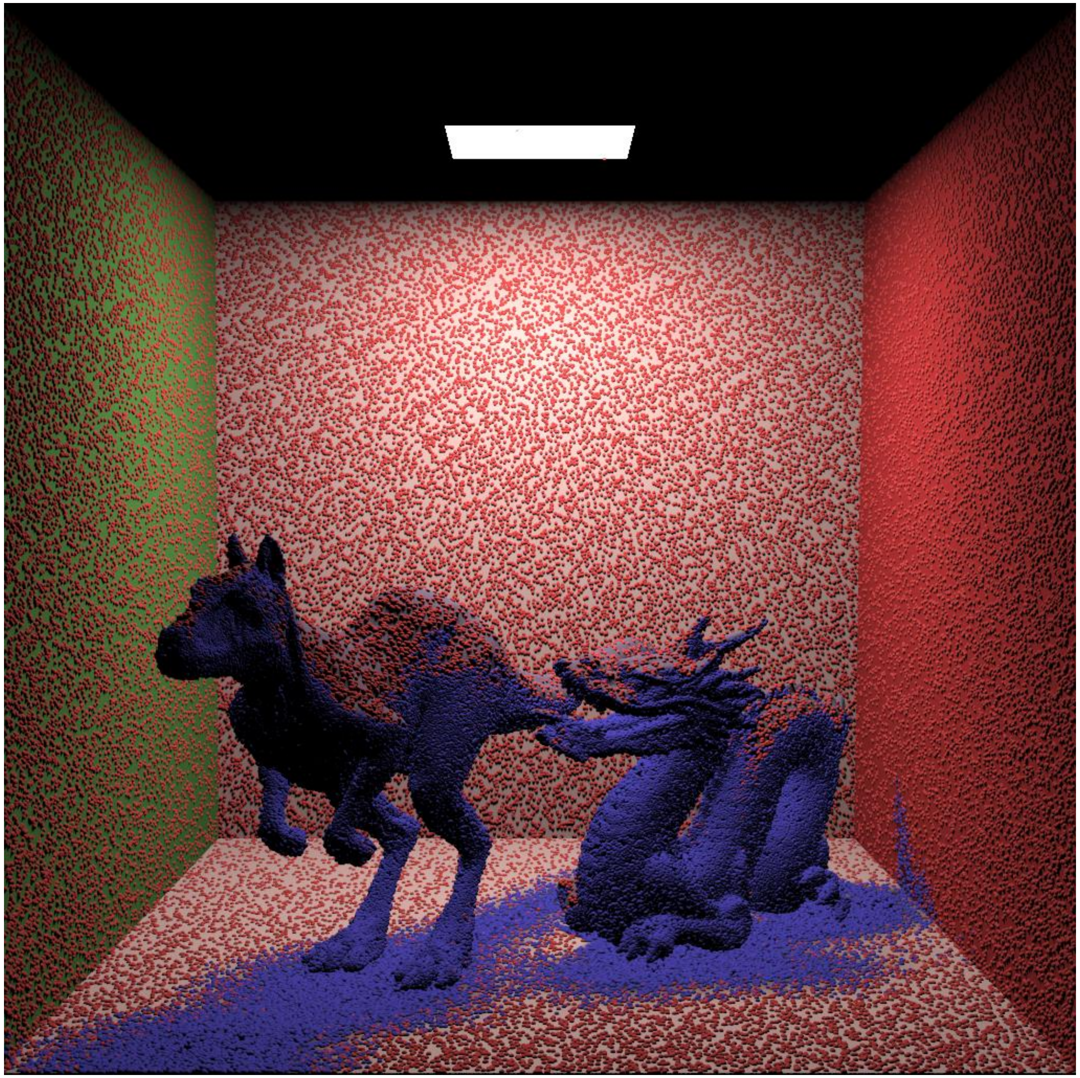
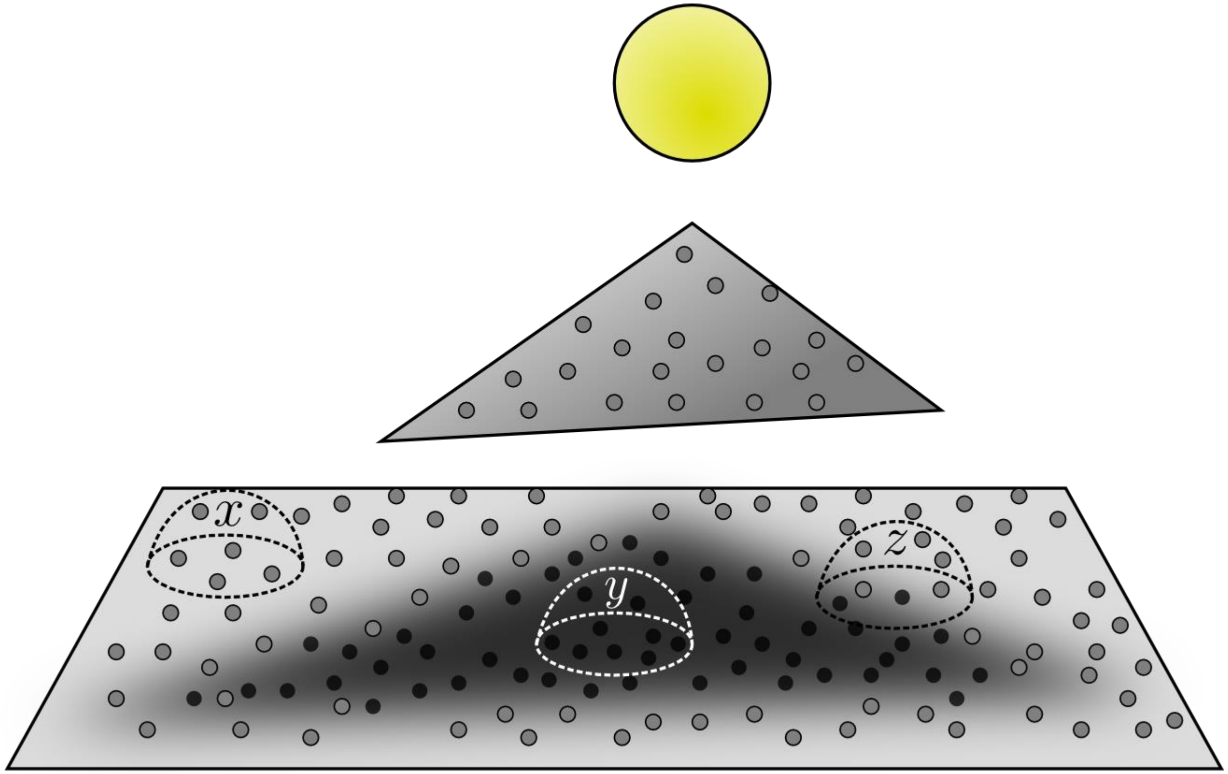


Phase 1: Construct Occlusion Map



Phase 2: Rendering

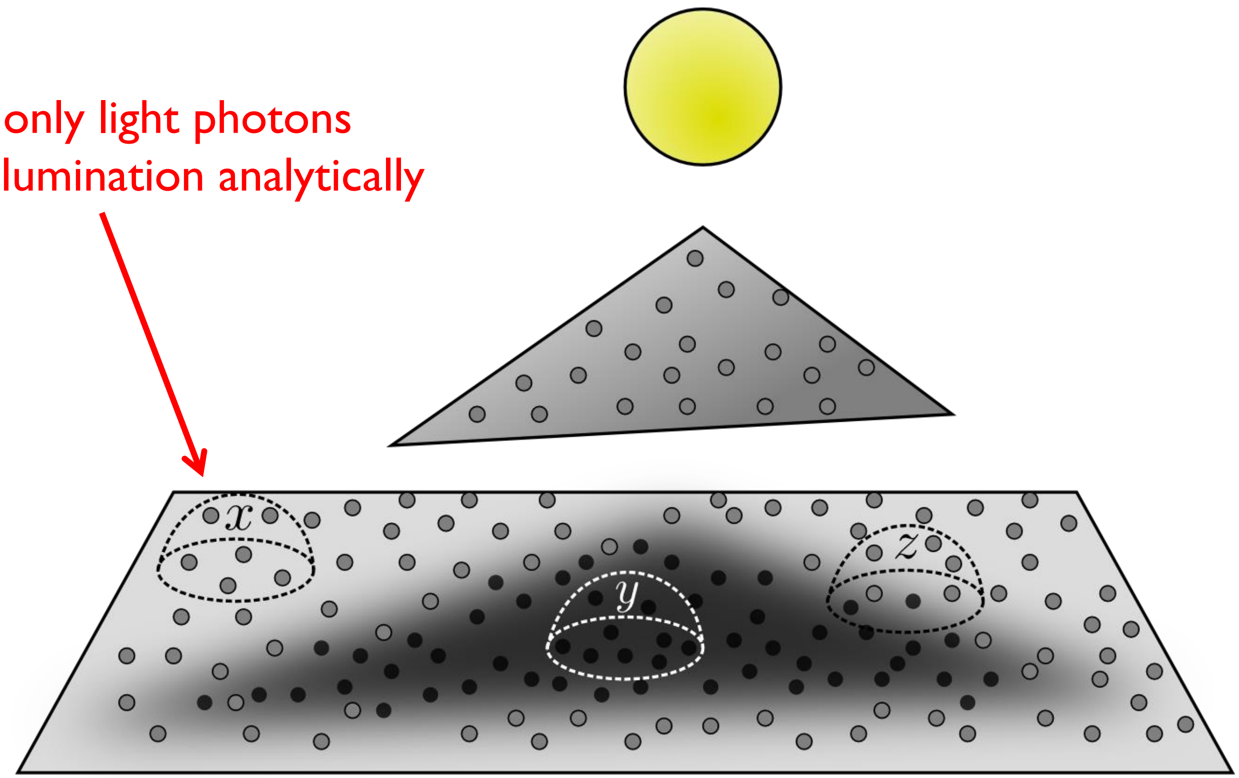
- For each point to be shaded:
  - Locate closest shadow and light photons



Phase 2: Rendering

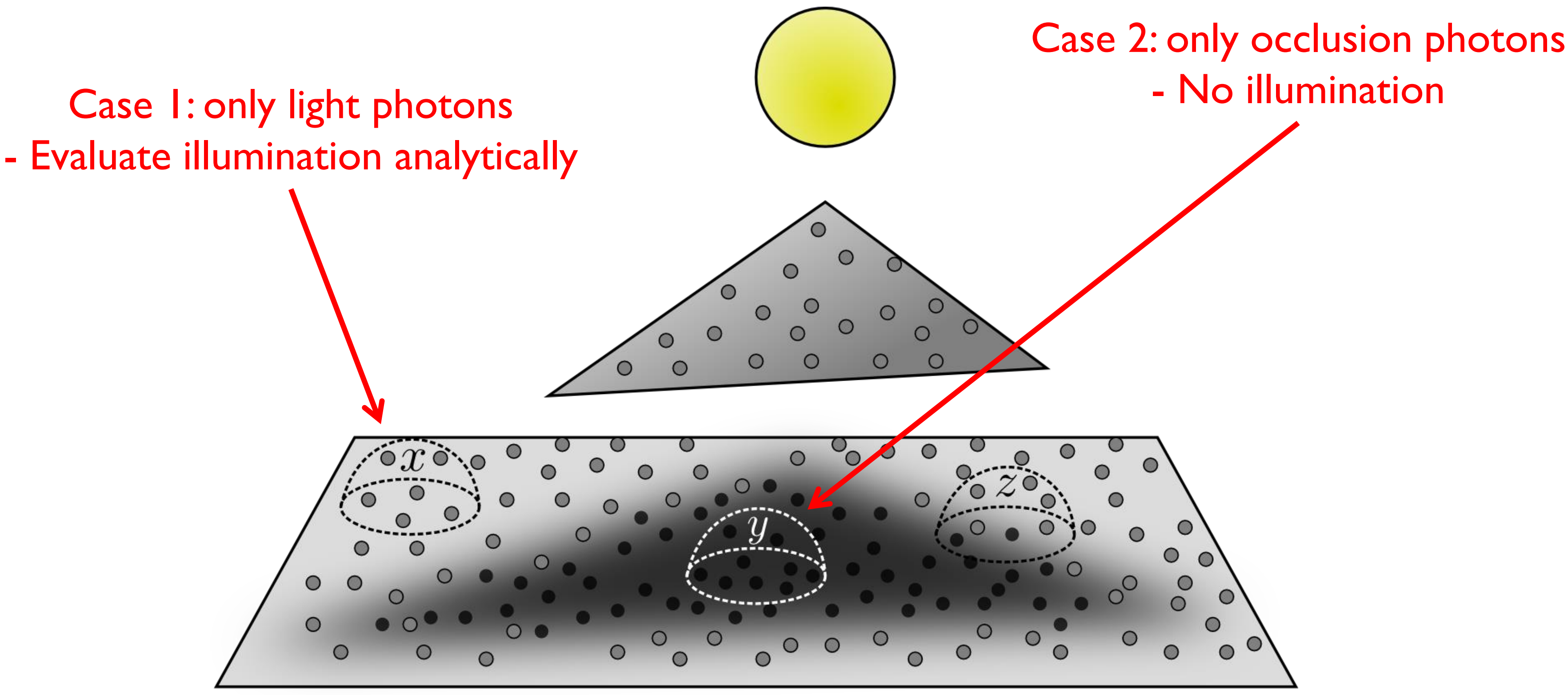
- For each point to be shaded:
  - Locate closest shadow and light photons

Case 1: only light photons
- Evaluate illumination analytically



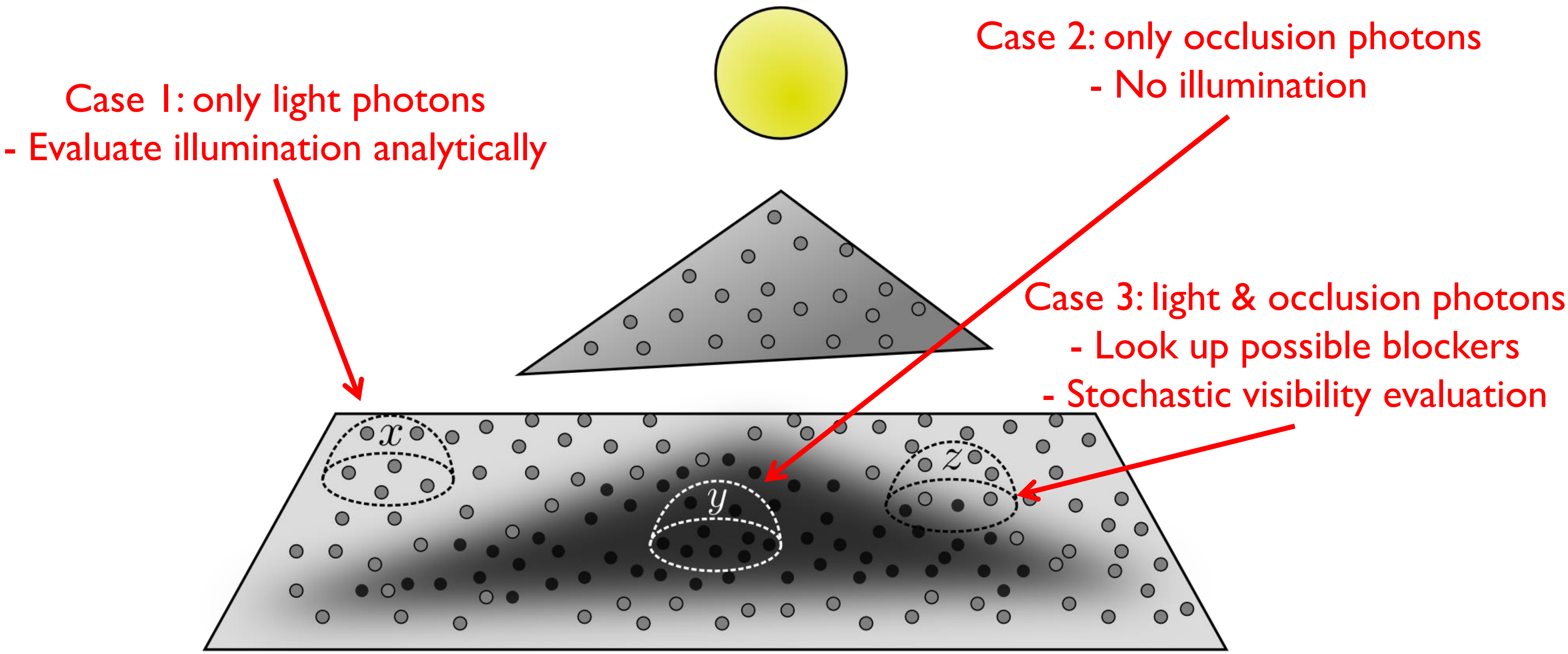
Phase 2: Rendering

- For each point to be shaded:
  - Locate closest shadow and light photons



Phase 2: Rendering

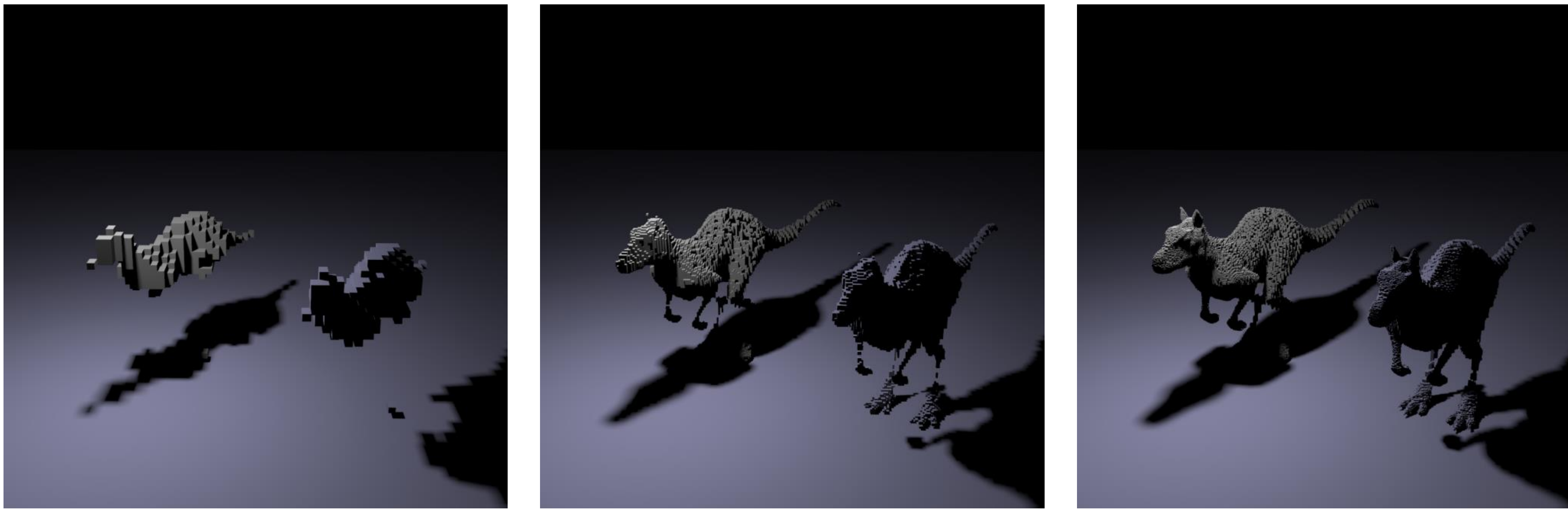
- For each point to be shaded:
  - Locate closest shadow and light photons



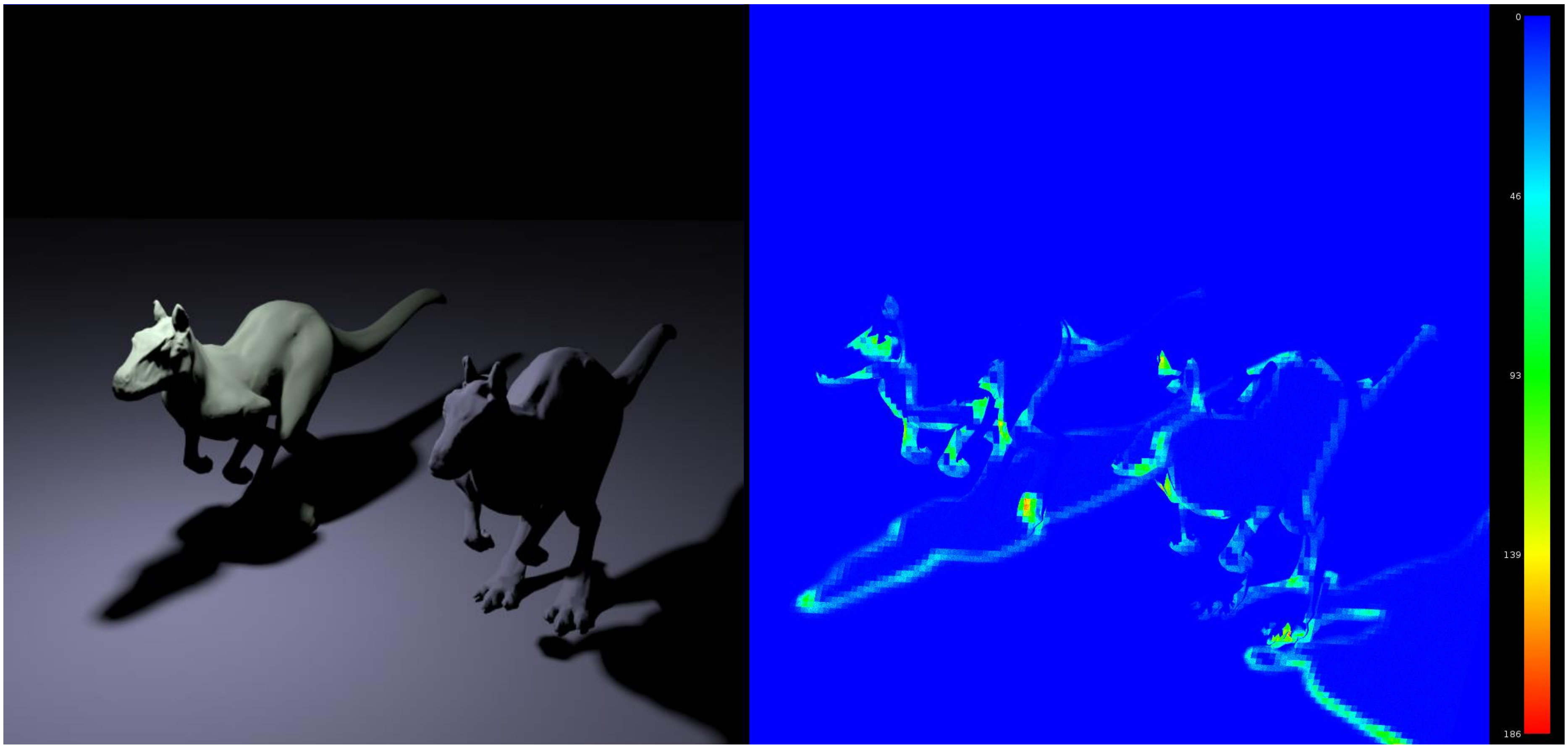
Stochastic evaluation of visibility

- Subdivision of possible blockers in two groups A and B:
  - Sort by subtended solid angle as seen from point to be shaded
  - Split in 2 groups, keeping accumulated solid angle roughly equal

Optimizations: Volumetric Occluders



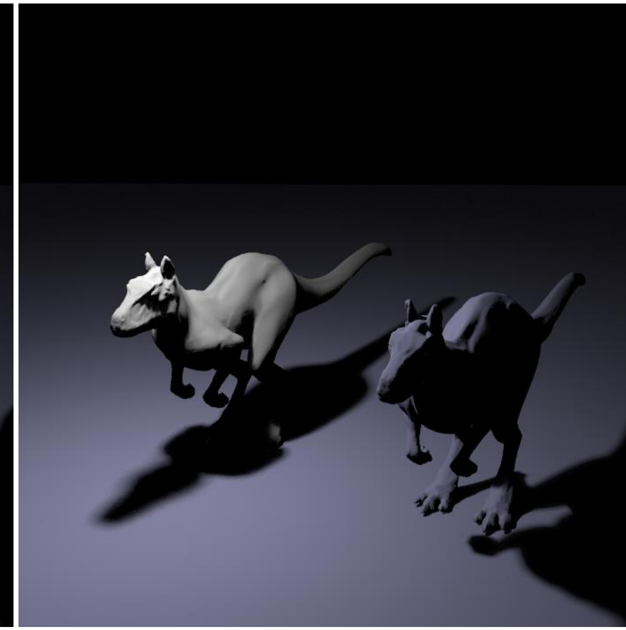
Optimizations: Adaptive occlusion photon generation



Results

Reference

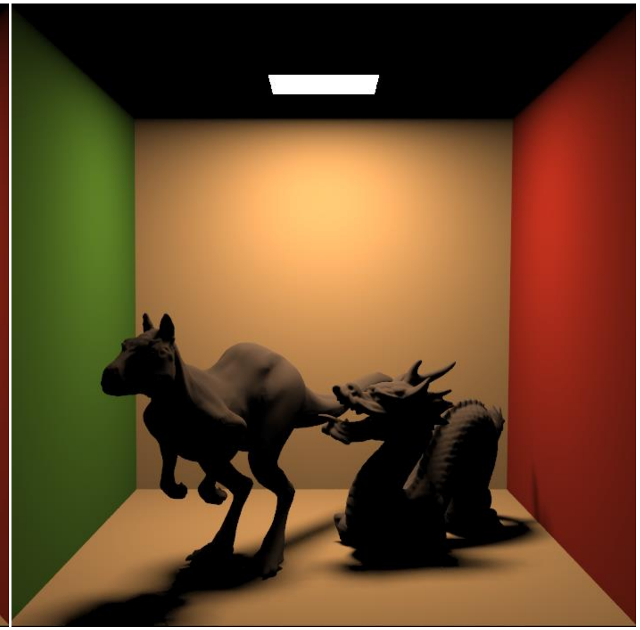
Occlusion Map w/o probvis
306 sec
27494M intersections



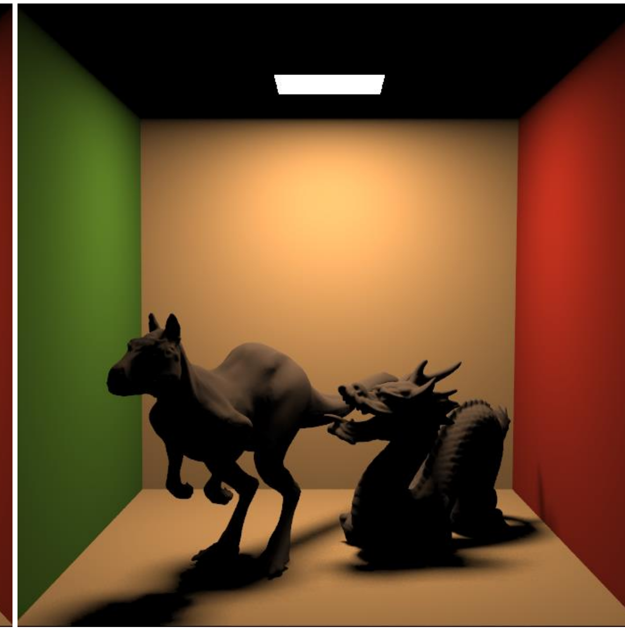
Occlusion Map with probvis
262 sec (-14.4%)
21407M intersections (-23.4%)

Results

Reference

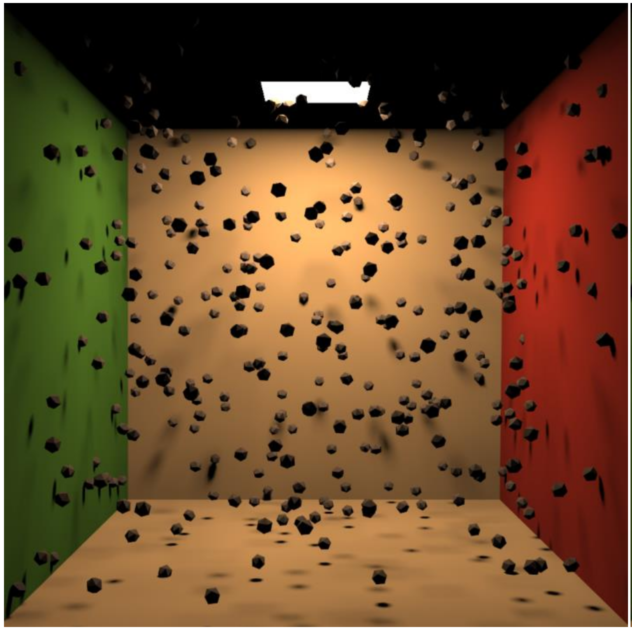


Occlusion Map w/o probvis
990 sec
96453M intersections

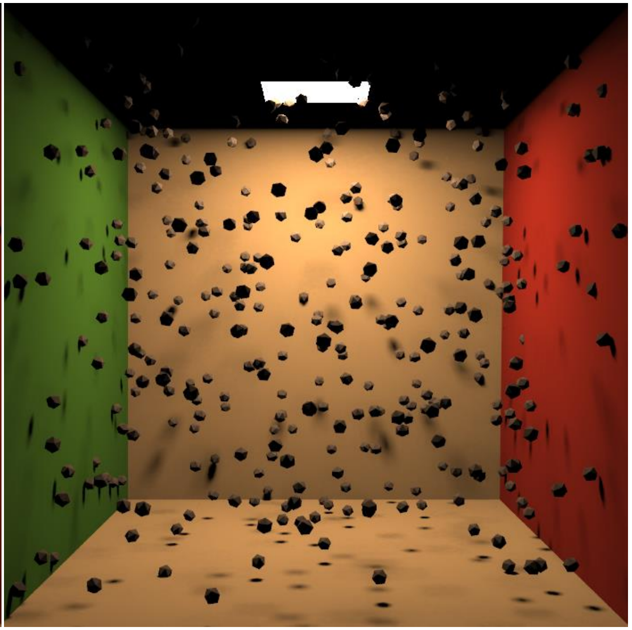


Occlusion Map with probvis
827 sec (-16.4%)
76283M intersections (-20.9%)

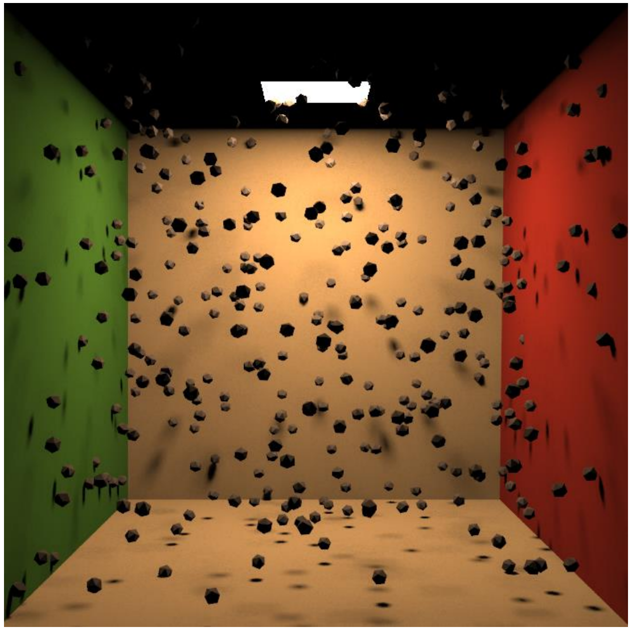
Results



Reference

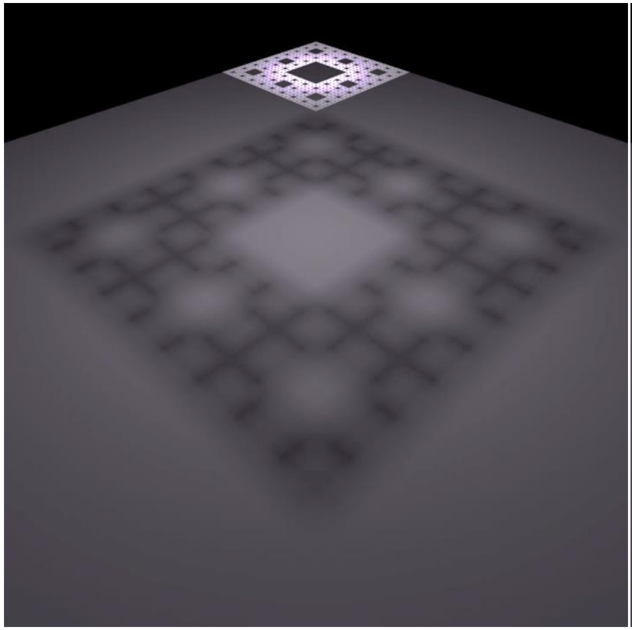


Occlusion Map w/o probvis
714 sec
61561 intersections

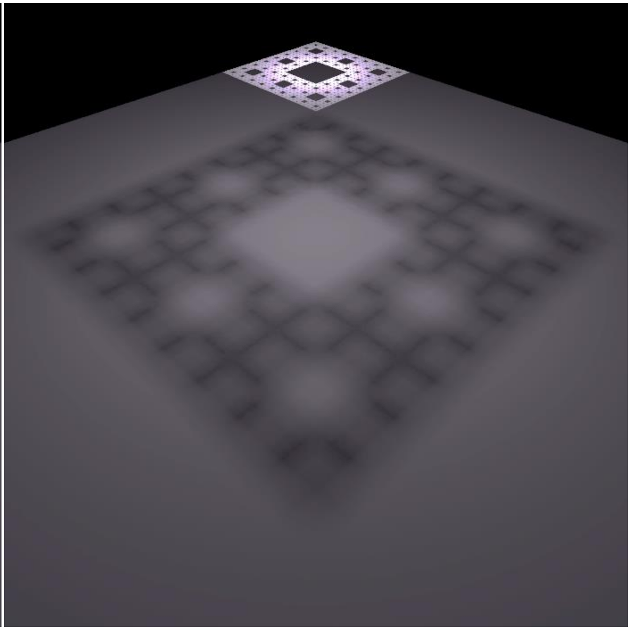


Occlusion Map with probvis
533 sec (-25.4%)
37913 intersections (-38.2%)

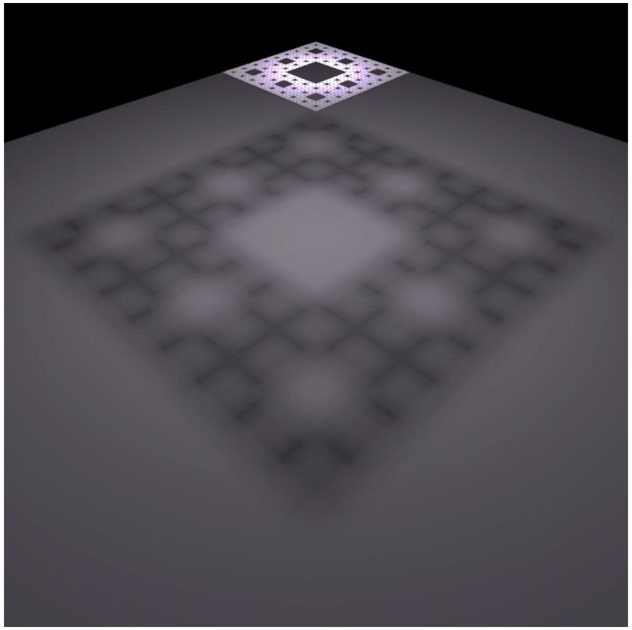
Results



Reference

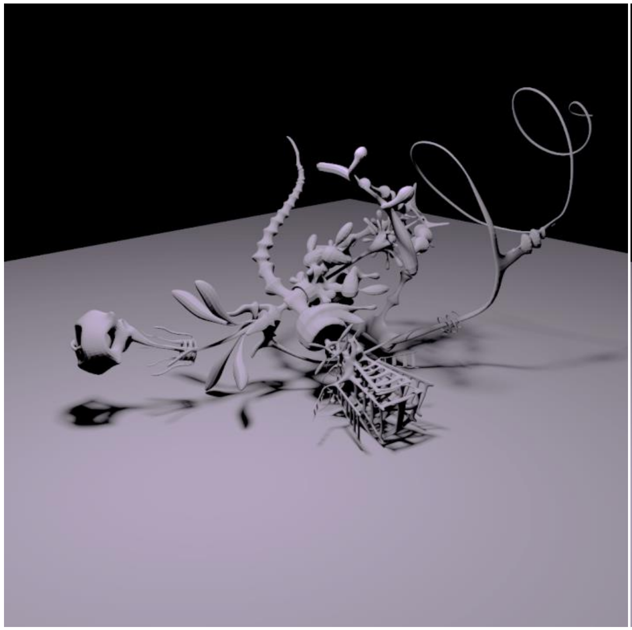


Occlusion Map w/o probvis
637 sec
63470M intersections

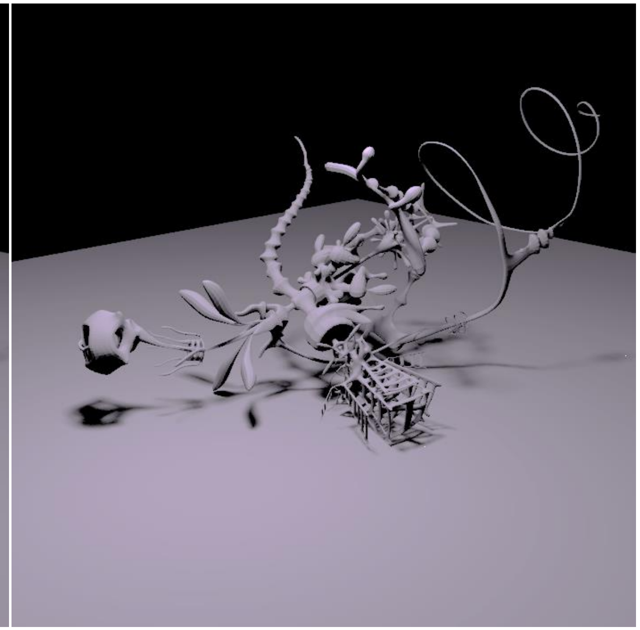


Occlusion Map with probvis
578 sec (-9.3%)
51927M intersections (-18.2%)

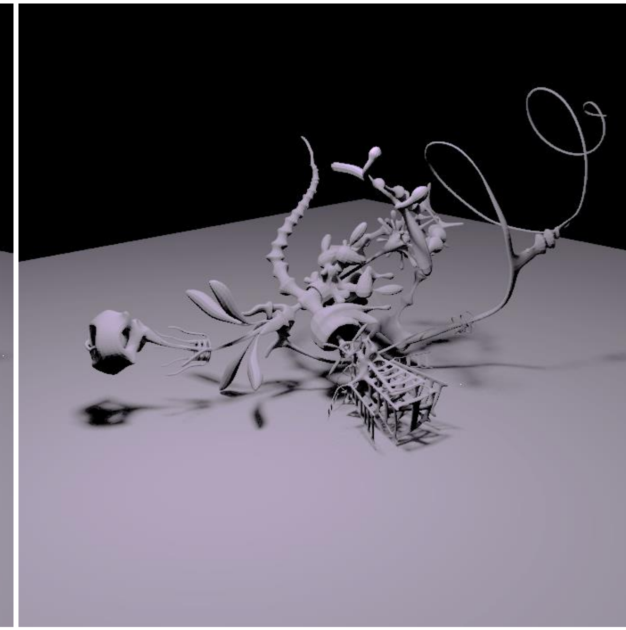
Results



Reference

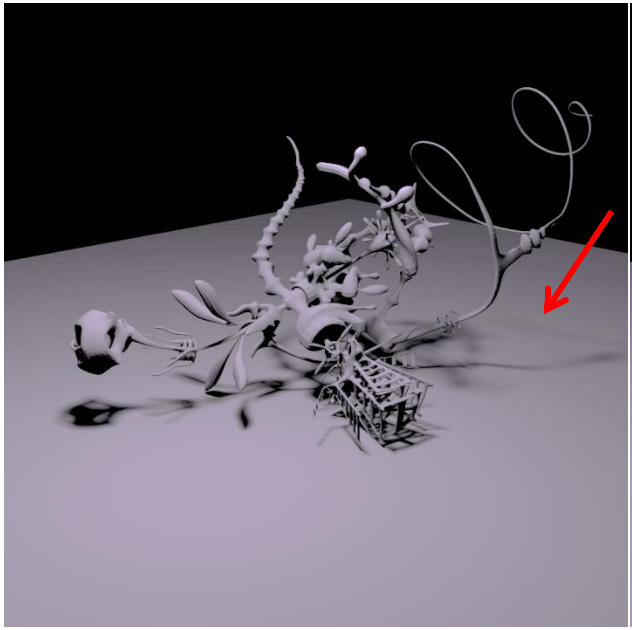


Occlusion Map w/o probvis
663 sec
70392M intersections

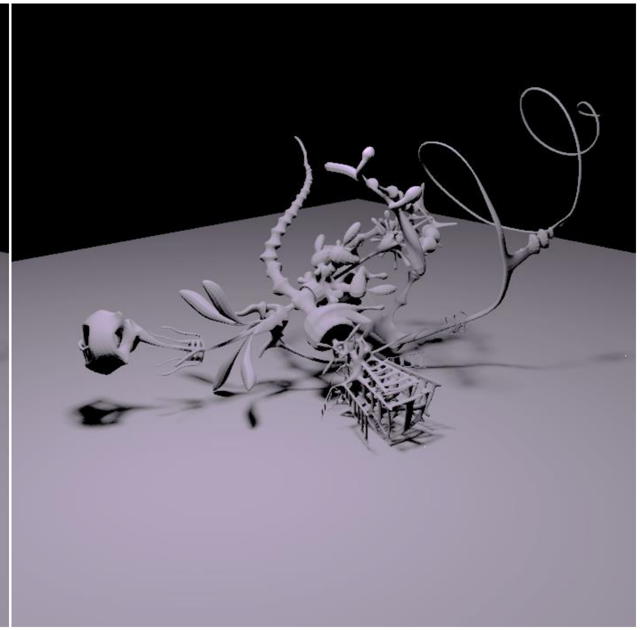


Occlusion Map with probvis
560 sec (-15.5%)
55203M intersections (-21.5%)

Results



Reference



Occlusion Map w/o probvis
663 sec
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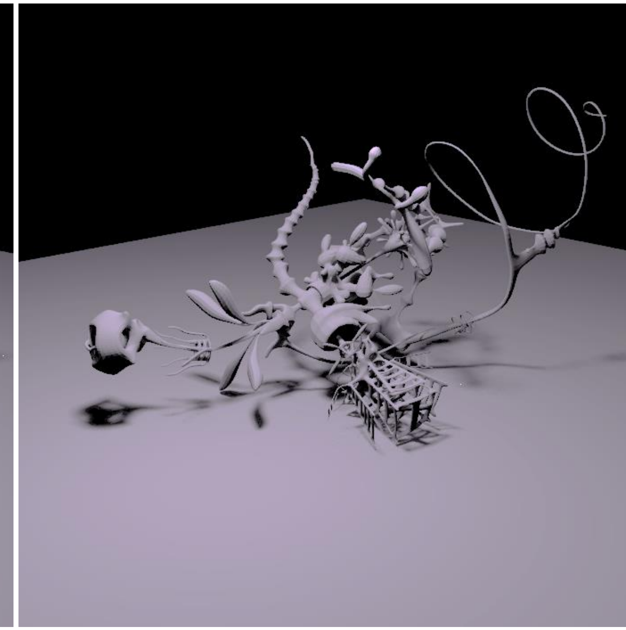

Occlusion Map with probvis
560 sec (-15.5%)
55203M intersections (-21.5%)

Results

Reference

Occlusion Map w/o probvis
628 sec
58449M intersections

Occlusion Map with probvis
588 sec (-6.3%)
44867M intersections (-23.2%)

Conclusion

- Stochastic evaluation of visibility
  - Unbiased images
- Room for reduction of number of intersection tests
  - Comparisons indicate reductions up to 38% are possible
  - New type of acceleration mechanism?
- New and experimental look at visibility
  - Will hopefully inspire future work!

Thank you!