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Probabilistic Visibility Estimation
using Geometry Proxies

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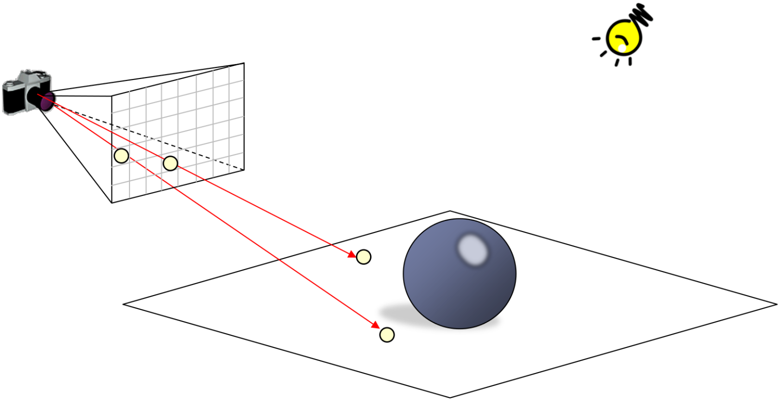
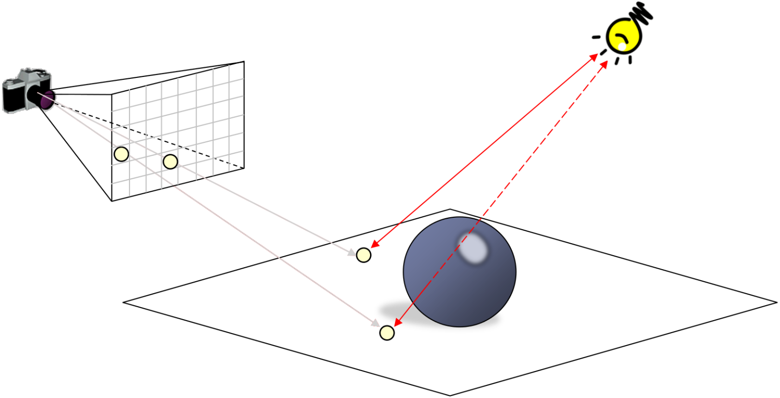
Visibility evaluations in graphics

"Find the first visible surface"



Visibility evaluations in graphics

"Test whether two points are mutually visible"



Motivation

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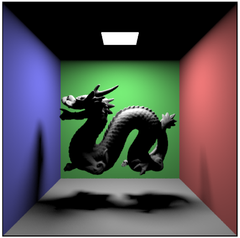
- **Exact visibility evaluation**
  slow, high quality shadows



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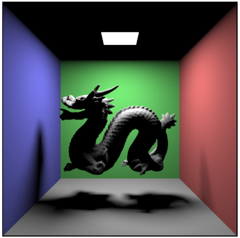
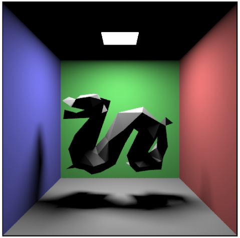


- **Approximate visibility evaluation**
  fast, low quality shadows



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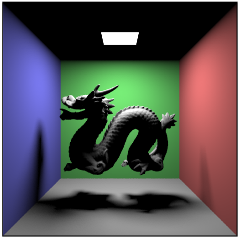

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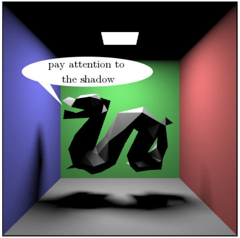

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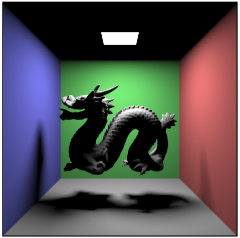

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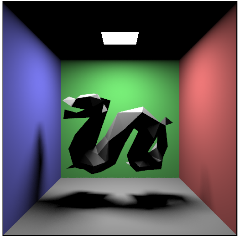

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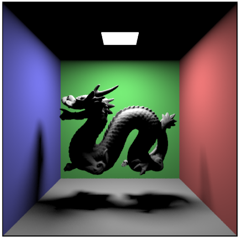

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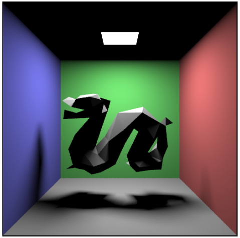

- Is it possible to render exact shadows using geometry proxies?

Related work

- **Approximate geometry**
  - Silvennoinen et al. [2014]: Occluder simplification using planar sections.
  - Décoret X et al. [2003]: Billboard clouds for extreme model simplification.
  - Heckbert et al. [1997]: Survey of polygonal surface simplification algorithms.
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Related work

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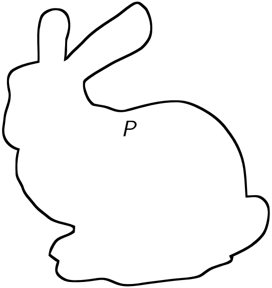
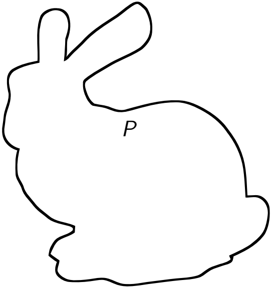
  - ...
- **Stochastic visibility evaluation**
  - Billen et al. [2013]: Probabilistic visibility estimation for direct illumination.

Probabilistic Visibility: Theory



Probabilistic Visibility: Theory

- $V_P\left(x, y\right)$: visibility of the original geometry.



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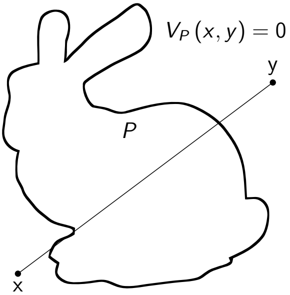

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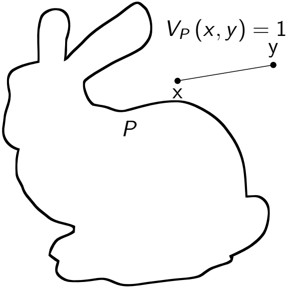

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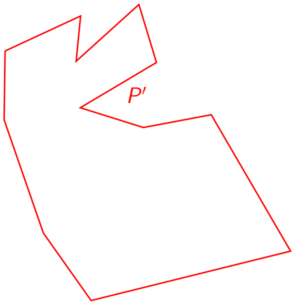

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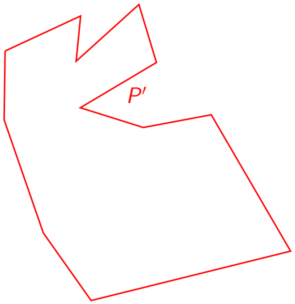
- $V_{P'}(x, y)$: visibility of the geometry proxy.



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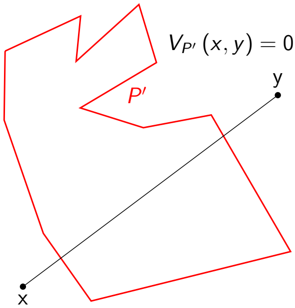

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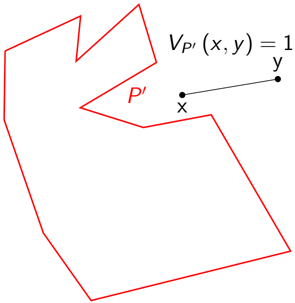
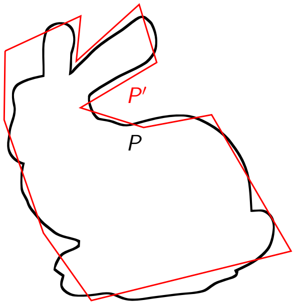


Probabilistic Visibility: Theory

- $V_P\,(x, y)$: visibility of the original geometry.
- $V_{P'}\,(x, y)$: visibility of the geometry proxy.
- In general:

$$V_P\,(x, y) \neq V_{P'}\,(x, y)$$



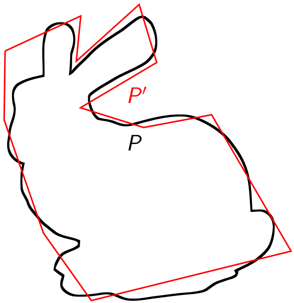
Probabilistic Visibility: Theory

- $V_P(x, y)$: visibility of the original geometry.
- $V_{P'}(x, y)$: visibility of the geometry proxy.
- In general:

$$V_P(x, y) \neq V_{P'}(x, y)$$

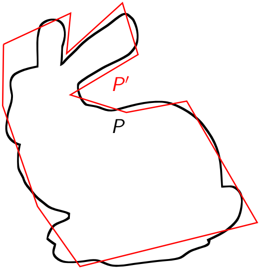
- Write the exact visibility as:

$$V_P(x, y) = V_{P'}(x, y) + c(x, y)$$



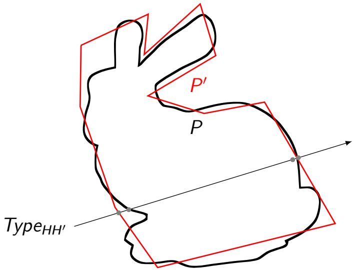
Probabilistic Visibility: Theory

	$V_P(x,y)$	$V_{P'}(x,y)$	$c(x,y)$



Probabilistic Visibility: Theory

	$V_P(x,y)$	$V_{P'}(x,y)$	$c(x,y)$
$Type_{HH'}$	0	0	



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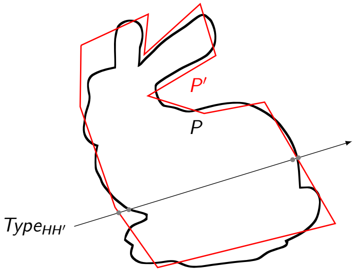
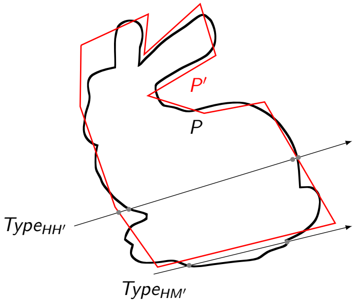


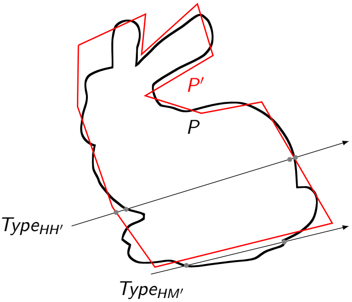
Probabilistic Visibility: Theory

	$V_P(x,y)$	$V_{P'}(x,y)$	$c(x,y)$
$Type_{HH'}$	0	0	0
$Type_{HM'}$	0	1	



Probabilistic Visibility: Theory

	$V_P(x,y)$	$V_{P'}(x,y)$	$c(x,y)$
$Type_{HH'}$	0	0	0
$Type_{HM'}$	0	1	-1



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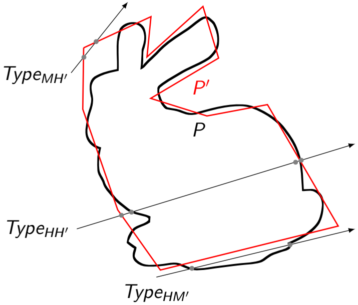
$Type_{MH'}$	1	0	



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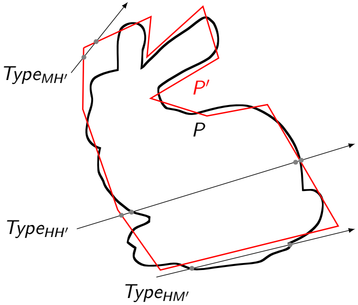

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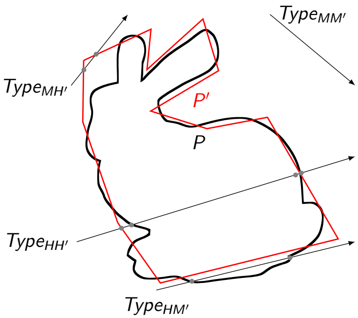

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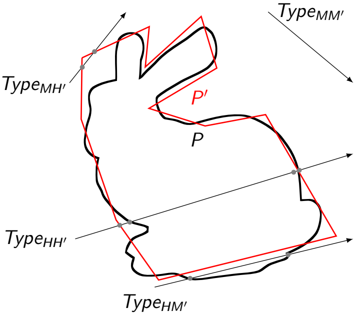

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$$V_P(x,y) = V_{P'}(x,y) + c(x,y)$$

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$$+V_P(x,y)\,\overline{V_{P'}(x,y)}$$

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Probabilistic Visibility: Theory

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- Monte Carlo estimation of a sum:

$$S = s_1 + s_2 + ... + s_n$$

Probabilistic Visibility: Theory

- Monte Carlo estimation of a sum:

$$S = s_1 + s_2 + ... + s_n$$

- Pick a single term s_i with probability p_i:

$$\tilde{S} = \frac{s_i}{p_i}$$

$$\tilde{S} = \frac{1}{M} \sum_{i=1}^{M} \frac{s_i}{p_i}$$

$$E\left[\tilde{S}\right] = S$$

Probabilistic Visibility: Theory

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$$V_P(x, y) = V_{P'}(x, y) + c(x, y)$$

Probabilistic Visibility: Theory

$$\begin{aligned} V_P(x,y) &= V_{P'}(x,y) + c(x,y) \\ &= V_{P'}(x,y) + V_P(x,y)\,\overline{V_{P'}(x,y)} - V_{P'}(x,y)\,\overline{V_P(x,y)} \end{aligned}$$

Probabilistic Visibility: Theory

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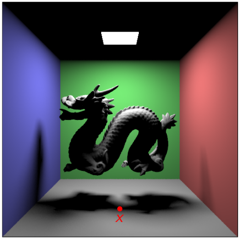
$$\tilde{V}(x,y) = \begin{cases} \dfrac{V_{P'}(x,y)}{p_{proxy}} & \text{with probability } p_{proxy} \\ \dfrac{V_P(x,y)\,\overline{V_{P'}(x,y)}}{p_{correction_+}} & \text{with probability } p_{correction_+} \\ -\dfrac{V_{P'}(x,y)\,\overline{V_P(x,y)}}{p_{correction_-}} & \text{with probability } p_{correction_-} \end{cases}$$

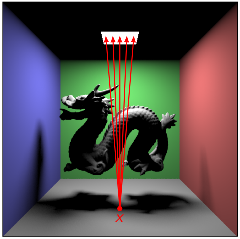
Probabilistic Visibility: Theory

$$L_{direct}(x \to \theta) = \int_A f(x, \overline{yx} \leftrightarrow \theta)\, L(y \to x)\, V_P(x, y)\, G(x, y)\, dA$$



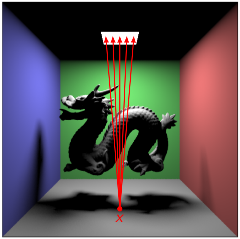
Probabilistic Visibility: Theory

$$L_{direct}\left(x \rightarrow \theta\right) = \int_A f\left(x, \overline{yx} \leftrightarrow \theta\right) L\left(y \rightarrow x\right) V_P\left(x, y\right) G\left(x, y\right) dA$$



Probabilistic Visibility: Theory

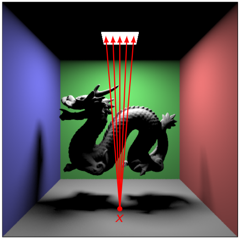
$$L_{direct}(x \to \theta) = \int_A f(x, \overline{yx} \leftrightarrow \theta) \, L(y \to x) \, \tilde{V}(x,y) \, G(x,y) \, dA$$



Probabilistic Visibility: Theory

$$L_{direct}(x \rightarrow \theta) =$$

$$\int_A f(x, \overline{yx} \leftrightarrow \theta)\, L(y \rightarrow x) \begin{cases} \frac{V_{P'}}{p_{proxy}}(x,y) \\ \frac{V_P(x,y)\overline{V_{P'}(x,y)}}{p_{correction_+}} \\ \frac{V_{P'}(x,y)\overline{V_P(x,y)}}{p_{correction_-}} \end{cases} G(x,y)\, dA$$



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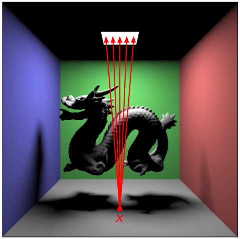

Three cases:

- with probability p_{proxy}, evaluate $\frac{V_{P'}(x,y)}{p_{proxy}}$

Probabilistic Visibility: Theory

$$L_{direct}(x \to \theta) =$$

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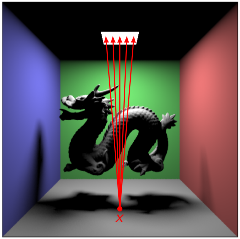
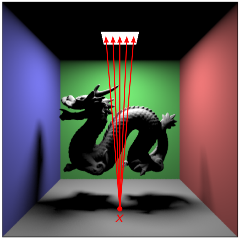


Three cases:

- with probability p_{proxy}, evaluate $\frac{V_{P'}(x,y)}{p_{proxy}}$
- with probability $p_{correction_+}$, evaluate $\frac{V_P(x,y)\overline{V_{P'}(x,y)}}{p_{correction_+}}$: first evaluate $V_{P'}(x,y)$:

Probabilistic Visibility: Theory

$$L_{direct}(x \to \theta) = \int_A f(x, \overline{yx} \leftrightarrow \theta)\, L(y \to x) \begin{cases} \frac{V_{P'}}{p_{proxy}}(x,y) \\ \frac{V_P(x,y)\overline{V_{P'}(x,y)}}{p_{correction_+}} \\ \frac{V_{P'}(x,y)\overline{V_P(x,y)}}{p_{correction\_}} \end{cases} G(x,y)\, dA$$



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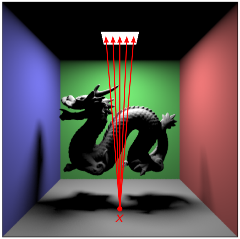
- with probability p_{proxy}, evaluate $\frac{V_{P'}(x,y)}{p_{proxy}}$
- with probability $p_{correction_+}$, evaluate $\frac{V_P(x,y)\overline{V_{P'}(x,y)}}{p_{correction_+}}$: first evaluate $V_{P'}(x,y)$:
  a. if $V_{P'}(x,y) = 1$: done

Probabilistic Visibility: Theory

$$L_{direct}(x \to \theta) =$$

$$\int_A f(x, \overline{yx} \leftrightarrow \theta)\, L(y \to x) \begin{cases} \frac{V_{P'}}{p_{proxy}}(x,y) \\ \frac{V_P(x,y)\overline{V_{P'}(x,y)}}{p_{correction_+}} \\ \frac{V_{P'}(x,y)\overline{V_P(x,y)}}{p_{correction_-}} \end{cases} G(x,y)\, dA$$



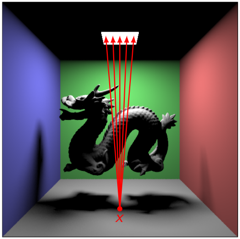
Three cases:

- with probability p_{proxy}, evaluate $\frac{V_{P'}(x,y)}{p_{proxy}}$
- with probability $p_{correction_+}$, evaluate $\frac{V_P(x,y)\overline{V_{P'}(x,y)}}{p_{correction_+}}$: first evaluate $V_{P'}(x,y)$:
  a. if $V_{P'}(x,y) = 1$: done
  b. if $V_{P'}(x,y) = 0$: evaluate $V_P(x,y)$

Probabilistic Visibility: Theory

$$L_{direct}(x \to \theta) =$$

$$\int_A f(x, \overline{yx} \leftrightarrow \theta)\, L(y \to x) \begin{cases} \frac{V_{P'}}{p_{proxy}}(x,y) \\ \frac{V_P(x,y)\overline{V_{P'}(x,y)}}{p_{correction_+}} \\ \frac{V_{P'}(x,y)\overline{V_P(x,y)}}{p_{correction_-}} \end{cases} G(x,y)\, dA$$



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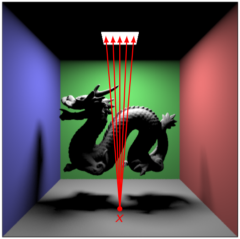
  b. if $V_{P'}(x,y) = 0$: evaluate $V_P(x,y)$
- with probability $p_{correction_-}$, evaluate $\frac{V_{P'}(x,y)\overline{V_P(x,y)}}{p_{correction_-}}$: first evaluate $V_{P'}(x,y)$:

Probabilistic Visibility: Theory

$$L_{direct}(x \rightarrow \theta) =$$

$$\int_A f(x, \overline{yx} \leftrightarrow \theta)\, L(y \rightarrow x) \begin{cases} \frac{V_{P'}}{p_{proxy}}(x,y) \\ \frac{V_P(x,y)\overline{V_{P'}(x,y)}}{p_{correction_+}} \\ \frac{V_{P'}(x,y)\overline{V_P(x,y)}}{p_{correction_-}} \end{cases} G(x,y)\, dA$$



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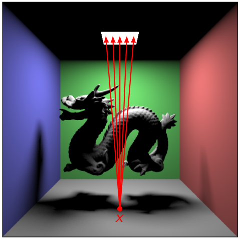
  a. if $V_{P'}(x,y) = 0$: done

Probabilistic Visibility: Theory

$$L_{direct}(x \to \theta) =$$

$$\int_A f(x, \overline{yx} \leftrightarrow \theta) L(y \to x) \begin{cases} \frac{V_{P'}}{p_{proxy}}(x,y) \\ \frac{V_P(x,y)\overline{V_{P'}(x,y)}}{p_{correction_+}} \\ \frac{V_{P'}(x,y)\overline{V_P(x,y)}}{p_{correction_-}} \end{cases} G(x,y)\, dA$$



Three cases:

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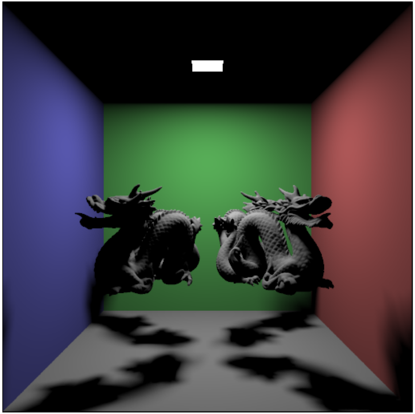
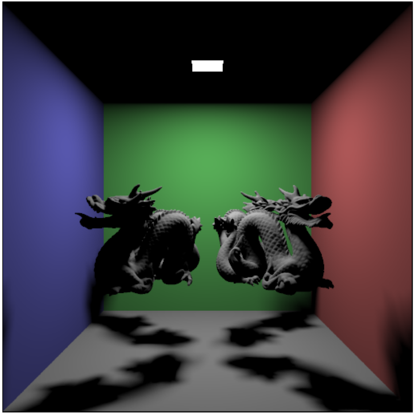
  a. if $V_{P'}(x,y) = 1$: done
  b. if $V_{P'}(x,y) = 0$: evaluate $V_P(x,y)$
- with probability $p_{correction_-}$, evaluate $\frac{V_{P'}(x,y)\overline{V_P(x,y)}}{p_{correction_-}}$: first evaluate $V_{P'}(x,y)$:
  a. if $V_{P'}(x,y) = 0$: done
  b. if $V_{P'}(x,y) = 1$: evaluate $\overline{V_P(x,y)}$

Results

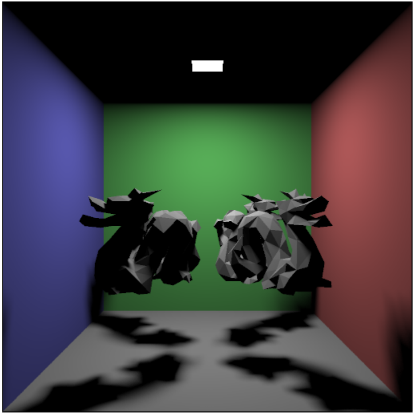


Test scene

Results

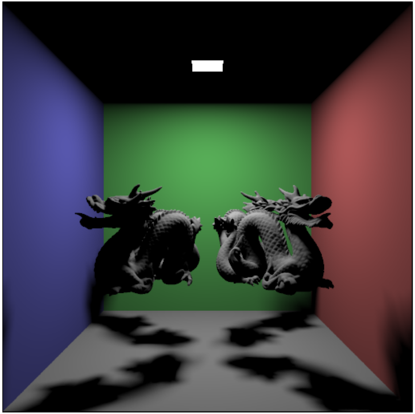


Test scene

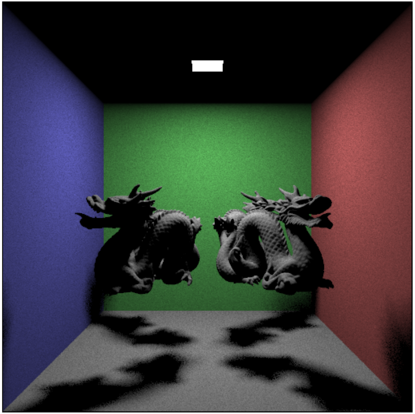


Geometry proxies used for stochastic visibility estimation

Results

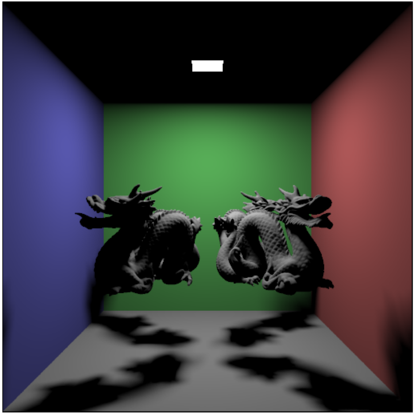


Ground truth

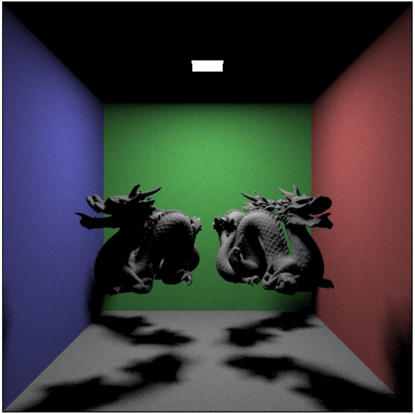


Probabilistic evaluation using 32 shadow rays

Results

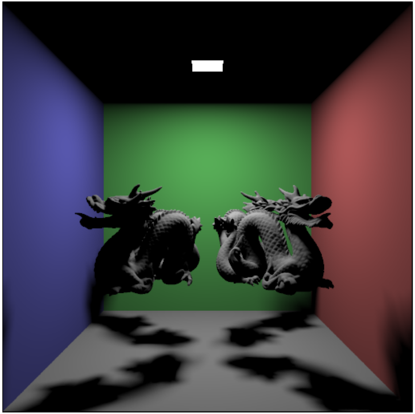


Ground truth

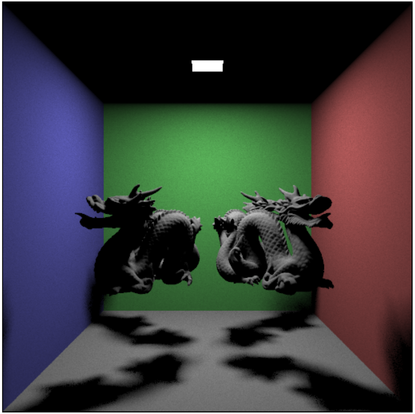


Probabilistic evaluation using 64 shadow rays

Results

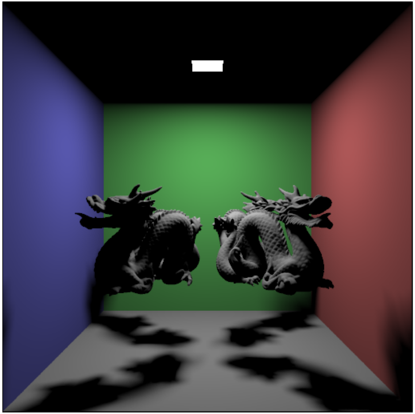


Ground truth

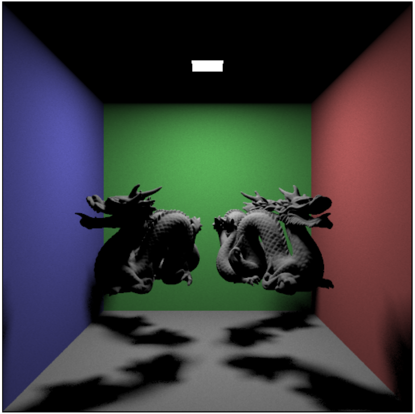


Probabilistic evaluation using 128 shadow rays

Results

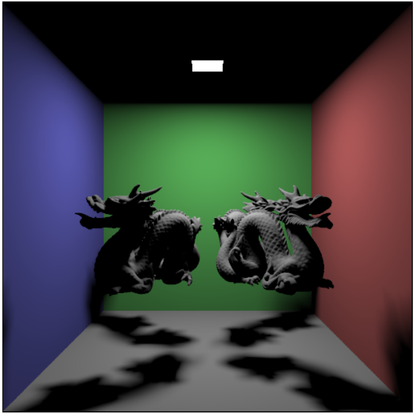


Ground truth

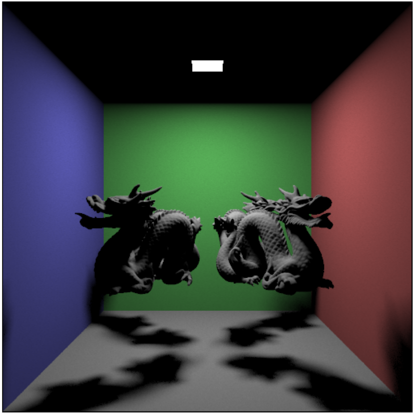


Probabilistic evaluation using 256 shadow rays

Results

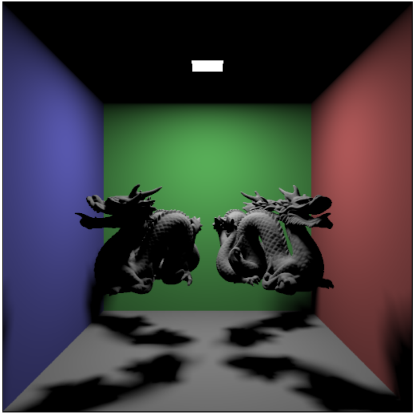


Ground truth

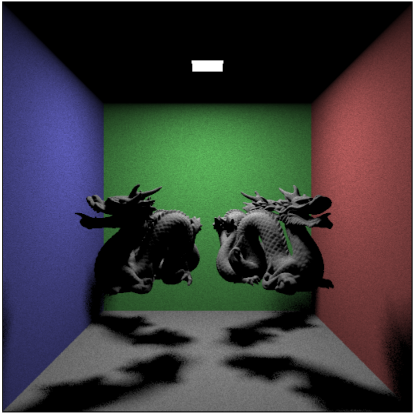


Probabilistic evaluation using 512 shadow rays

Results

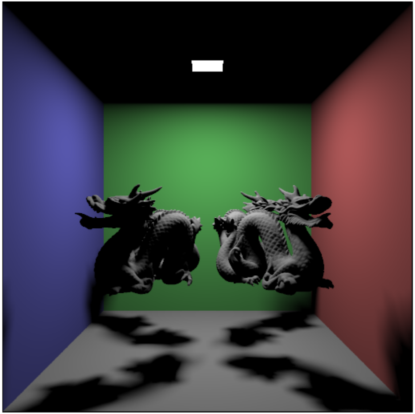


Ground truth

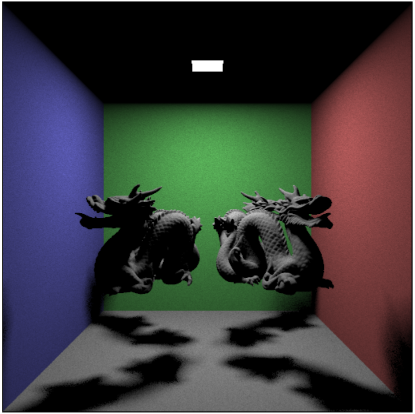


Probabilistic evaluation using 32 shadow rays

Results

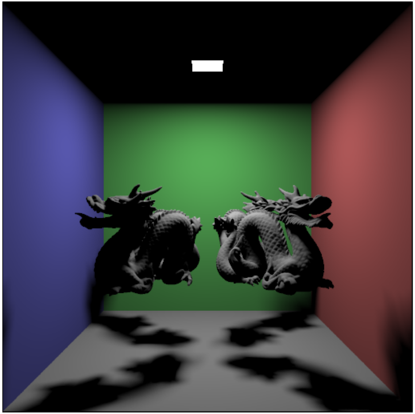


Ground truth

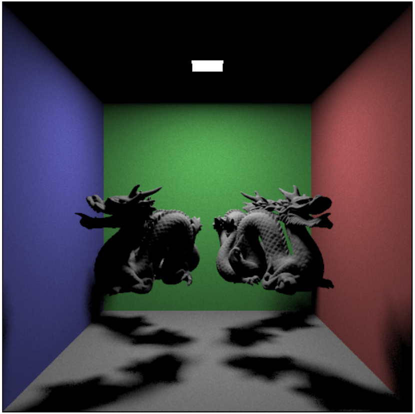


Probabilistic evaluation using 64 shadow rays

Results

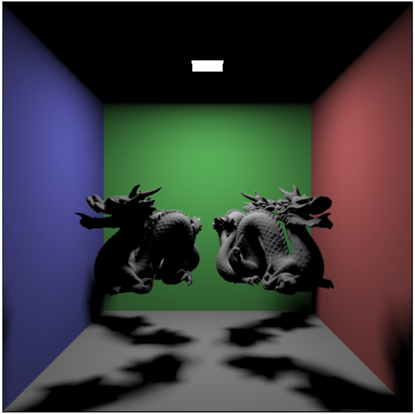


Ground truth

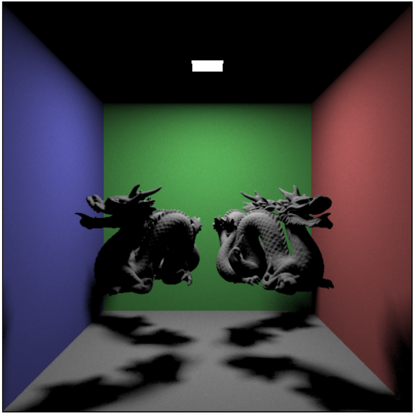


Probabilistic evaluation using 128 shadow rays

Results

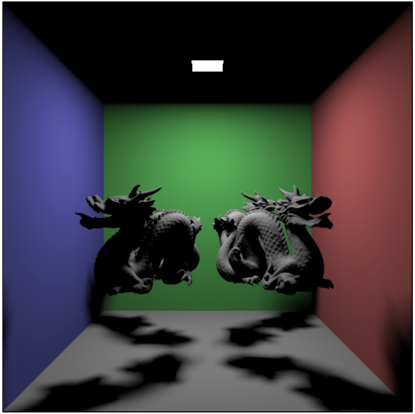


Ground truth

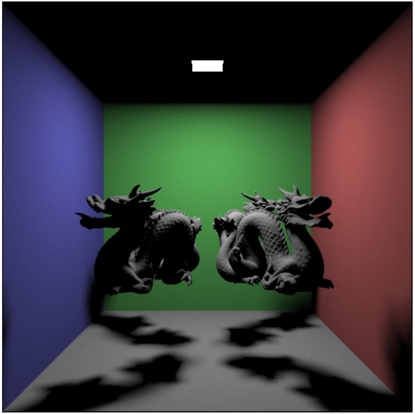


Probabilistic evaluation using 256 shadow rays

Results



Ground truth



Probabilistic evaluation using 512 shadow rays

Practical algorithm

- **Goals**

Practical algorithm

- **Goals**
  - Minimize the variance

Practical algorithm

- **Goals**
  - Minimize the variance
  - Account for cost

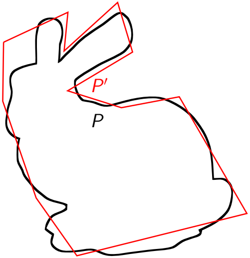
Practical algorithm

- **Goals**
  - Minimize the variance
  - Account for cost
- **How?**

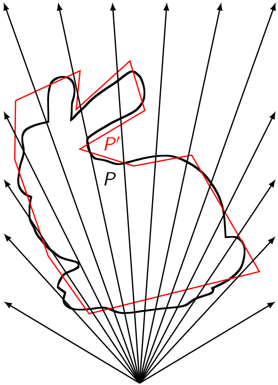
Practical algorithm

- **Goals**
  - Minimize the variance
  - Account for cost
- **How?**
  - Determine the optimal probabilities p_{proxy}, $p_{correction_+}$ and $p_{correction_-}$

Minimizing variance



Minimizing variance

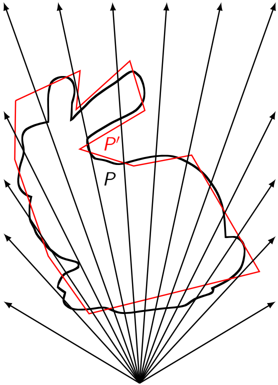


Minimizing variance

$$Var\left[\tilde{V}_P(x,y)\right] =$$

$$\frac{(1-V_{avg})(1-f_{hit})\left(p_{proxy}+p_{correction_+}\right)}{p_{proxy}\,p_{correction_+}} + \frac{V_{avg}(1-f_{miss})}{p_{correction_+}} + \frac{V_{avg}f_{miss}}{p_{proxy}} - V_{avg}^2$$

(derivation see paper)

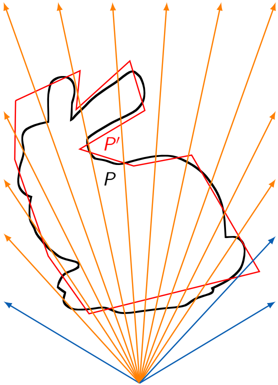


Minimizing variance

$$Var\left[\tilde{V}_P(x,y)\right] = \frac{(1-V_{avg})(1-f_{hit})\left(p_{proxy}+p_{correction_+}\right)}{p_{proxy}\,p_{correction_+}} + \frac{V_{avg}(1-f_{miss})}{p_{correction_+}} + \frac{V_{avg} f_{miss}}{p_{proxy}} - V_{avg}^2$$

(derivation see paper)

$$V_{avg} = \frac{\text{\#rays which miss P}}{\text{\#rays which miss P} + \text{\#rays which hit P}}$$



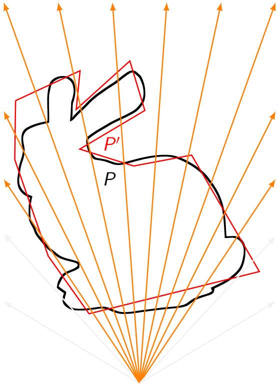
Minimizing variance

$$Var\left[\tilde{V}_P(x,y)\right] = \frac{(1-V_{avg})(1-f_{hit})\left(p_{proxy}+p_{correction_+}\right)}{p_{proxy}\,p_{correction_+}} + \frac{V_{avg}(1-f_{miss})}{p_{correction_+}} + \frac{V_{avg}f_{miss}}{p_{proxy}} - V_{avg}^2$$

(derivation see paper)

$$V_{avg} = \frac{\text{\#rays which miss P}}{\text{\#rays which miss P} + \text{\#rays which hit P}}$$

$$f_{hit} = \frac{\text{\#rays which hit both model and proxy}}{\text{\#rays which hit the model}}$$



Minimizing variance

$$Var\left[\tilde{V}_P(x,y)\right] = \frac{(1-V_{avg})(1-f_{hit})\left(p_{proxy}+p_{correction_+}\right)}{p_{proxy}\,p_{correction_+}} + \frac{V_{avg}(1-f_{miss})}{p_{correction_+}} + \frac{V_{avg}f_{miss}}{p_{proxy}} - V_{avg}^2$$

(derivation see paper)

$$V_{avg} = \frac{\#\text{rays which miss P}}{\#\text{rays which miss P} + \#\text{rays which hit P}}$$

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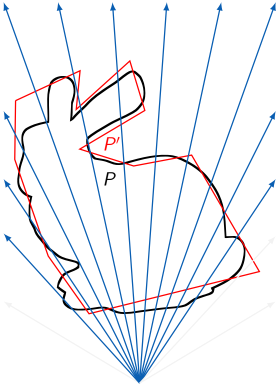


Minimizing variance

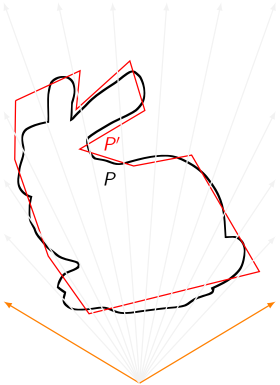
$$Var\left[\tilde{V}_P(x,y)\right] = \frac{(1-V_{avg})(1-f_{hit})\left(p_{proxy}+p_{correction_+}\right)}{p_{proxy}\,p_{correction_+}} + \frac{V_{avg}(1-f_{miss})}{p_{correction_+}} + \frac{V_{avg}f_{miss}}{p_{proxy}} - V_{avg}^2$$

(derivation see paper)

$$V_{avg} = \frac{\#\text{rays which miss P}}{\#\text{rays which miss P} + \#\text{rays which hit P}}$$

$$f_{hit} = \frac{\#\text{rays which hit both model and proxy}}{\#\text{rays which hit the model}}$$

$$f_{miss} = \frac{\#\text{rays which miss both model and proxy}}{\#\text{rays which miss the model}}$$



Minimizing variance

$$Var\left[\tilde{V}_P(x,y)\right] = \frac{(1-V_{avg})(1-f_{hit})\left(p_{proxy}+p_{correction_+}\right)}{p_{proxy}\,p_{correction_+}} + \frac{V_{avg}(1-f_{miss})}{p_{correction_+}} + \frac{V_{avg}f_{miss}}{p_{proxy}} - V_{avg}^2$$

(derivation see paper)

$$V_{avg} = \frac{\text{\#rays which miss P}}{\text{\#rays which miss P} + \text{\#rays which hit P}}$$

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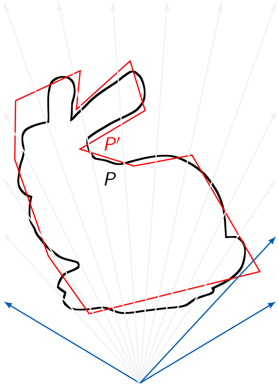

Minimizing variance

$$Var\left[\tilde{V}_P(x,y)\right] = \frac{(1-V_{avg})(1-f_{hit})\left(p_{proxy}+p_{correction_+}\right)}{p_{proxy}\,p_{correction_+}} + \frac{V_{avg}(1-f_{miss})}{p_{correction_+}} + \frac{V_{avg} f_{miss}}{p_{proxy}} - V_{avg}^2$$

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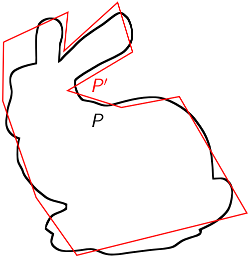


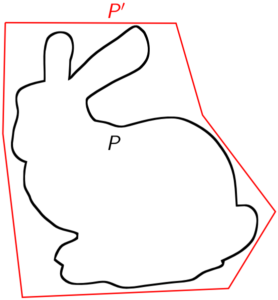
$$V_{avg} = \frac{\text{\#rays which miss P}}{\text{\#rays which miss P} + \text{\#rays which hit P}}$$

$$f_{hit} = \frac{\text{\#rays which hit both model and proxy}}{\text{\#rays which hit the model}}$$

$$f_{miss} = \frac{\text{\#rays which miss both model and proxy}}{\text{\#rays which miss the model}}$$

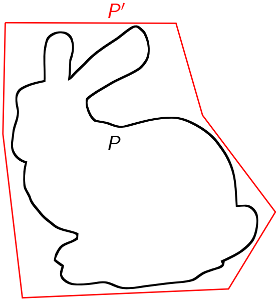
Minimizing the variance results in optimal probabilities p_{proxy}, $p_{correction_+}$ and $p_{correction_-}$.

Minimizing variance: Outside proxies



Minimizing variance: Outside proxies

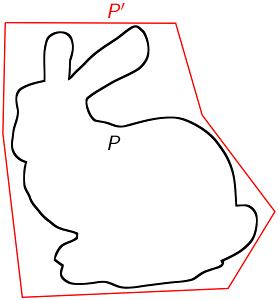
- Rays of type $Type_{HM'}$ do not exist.



Minimizing variance: Outside proxies

- Rays of type $Type_{HM'}$ do not exist.
- Estimator:

$$\tilde{V}_P(x,y) = \begin{cases} \frac{V_{P'}(x,y)}{p_{proxy}} & \text{probability } p_{proxy} \\ \frac{V_P(x,y)\overline{V_{P'}(x,y)}}{1-p_{proxy}} & \text{probability } 1-p_{proxy} \end{cases}$$

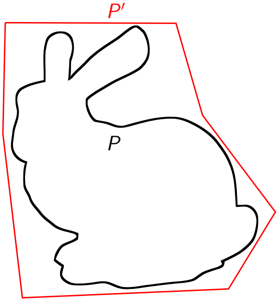


Minimizing variance: Outside proxies

- Rays of type $Type_{HM'}$ do not exist.
- Estimator:

$$\tilde{V}_P(x,y) = \begin{cases} \frac{V_{P'}(x,y)}{p_{proxy}} & \text{probability } p_{proxy} \\ \frac{V_P(x,y)\overline{V_{P'}(x,y)}}{1-p_{proxy}} & \text{probability } 1-p_{proxy} \end{cases}$$

- $f_{hit} = \frac{\#\text{rays which hit model and proxy}}{\#\text{rays which hit the model}} = 1$



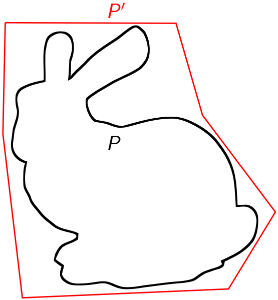
Minimizing variance: Outside proxies

- Rays of type $Type_{HM'}$ do not exist.
- Estimator:

$$\tilde{V}_P(x,y) = \begin{cases} \frac{V_{P'}(x,y)}{p_{proxy}} & \text{probability } p_{proxy} \\ \frac{V_P(x,y)\overline{V_{P'}(x,y)}}{1-p_{proxy}} & \text{probability } 1 - p_{proxy} \end{cases}$$

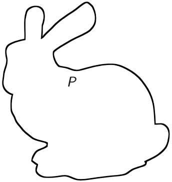
- $f_{hit} = \frac{\#\text{rays which hit model and proxy}}{\#\text{rays which hit the model}} = 1$
- Variance:

$$\frac{V_{avg}(1 - f_{miss})}{1 - p_{proxy}} + \frac{V_{avg} f_{miss}}{p_{proxy}} - V_{avg}^2$$



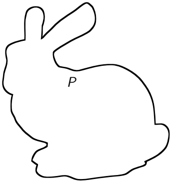
Minimizing variance: Outside proxies

- $Var\left[\tilde{V}\left(x,y\right)\right] = \frac{V_{avg}(1-f_{miss})}{1-p_{proxy}} + \frac{V_{avg}f_{miss}}{p_{proxy}} - V_{avg}^2$



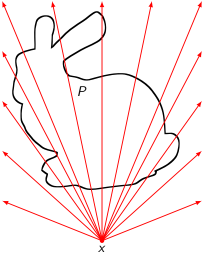
Minimizing variance: Outside proxies

- $Var\left[\tilde{V}(x,y)\right] = \frac{V_{avg}(1-f_{miss})}{1-p_{proxy}} + \frac{V_{avg} f_{miss}}{p_{proxy}} - V_{avg}^2$
- $f_{miss} = \frac{\text{\#rays which miss both model and proxy}}{\text{\#rays which miss the model}}$



Minimizing variance: Outside proxies

- $Var\left[\tilde{V}(x,y)\right] = \frac{V_{avg}(1-f_{miss})}{1-p_{proxy}} + \frac{V_{avg} f_{miss}}{p_{proxy}} - V_{avg}^2$
- $f_{miss} = \frac{\text{\#rays which miss both model and proxy}}{\text{\#rays which miss the model}}$



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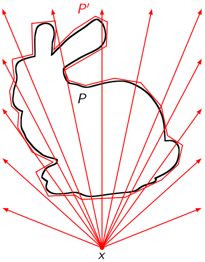

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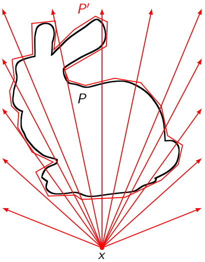
  $\cong 1$



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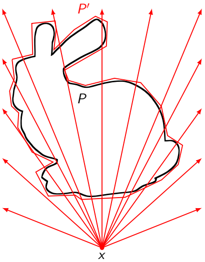
  $\cong 1$

  $\Rightarrow Var\left[\tilde{V}(x,y)\right] \simeq \frac{V_{avg}}{p_{proxy}} - V_{avg}^2$



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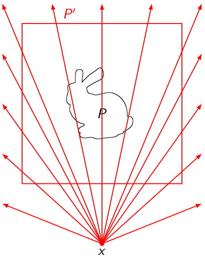

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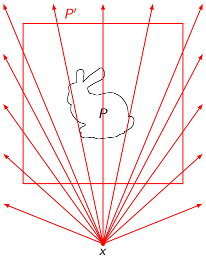

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  $\cong 0$

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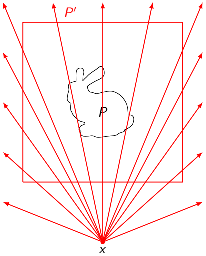
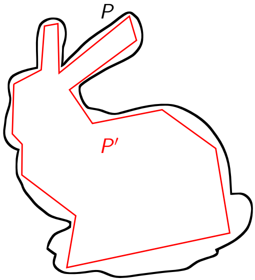


Variance: Inside proxies

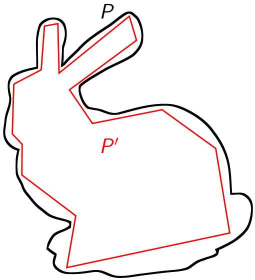
- Rays of type $Type_{MH'}$ do not exist.



Variance: Inside proxies

- Rays of type $Type_{MH'}$ do not exist.
- Estimator:

$$\tilde{V_P}(x,y) = \begin{cases} \frac{V_{P'}(x,y)}{p_{proxy}} & \text{probability } p_{proxy} \\ -\frac{V_{P'}(x,y)\overline{V_P(x,y)}}{1-p_{proxy}} & \text{probability } 1 - p_{proxy} \end{cases}$$



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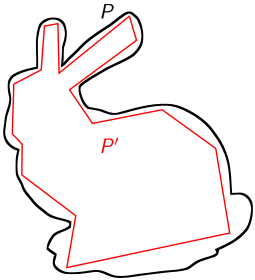

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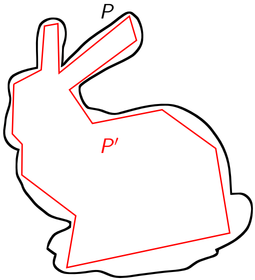
- $f_{miss} = \frac{\text{\#rays which miss model and proxy}}{\text{\#rays which miss the model}} = 1$
- Variance:

$$\frac{(1-V_{avg})(1-f_{hit})}{p_1(1-p_{proxy})} + \frac{V_{avg}}{p_{proxy}} - V_{avg}{}^2$$



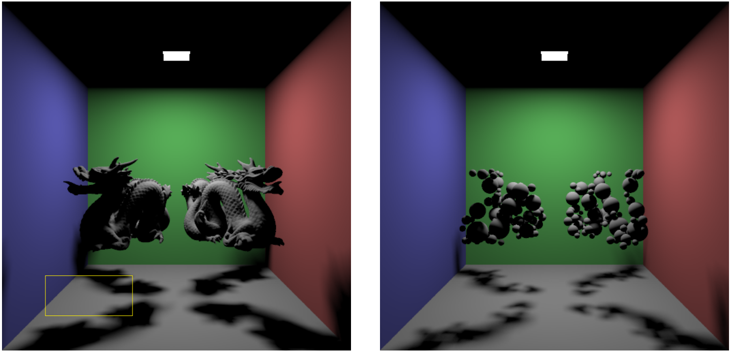
Minimum variance: Results

- **Problem:**
  optimal probabilities contain unknown parameters V_{avg}, f_{hit}, f_{miss}

Minimum variance: Results

- **Problem:**
  optimal probabilities contain unknown parameters V_{avg}, f_{hit}, f_{miss}
- **Solution:**
  estimate unknown parameters for a shading point x using probe rays.

Minimum variance: Inside proxies

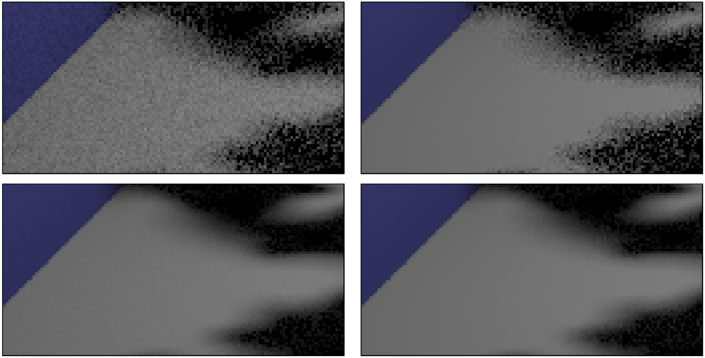


Minimum variance: Inside proxies

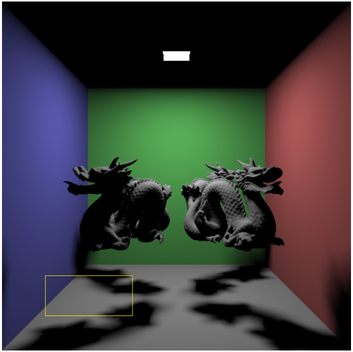
Inside proxy equal probabilities | **Inside proxy** optimal probabilities

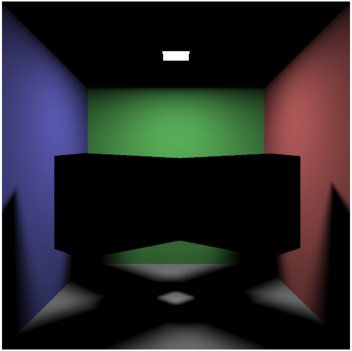
16sr

1024sr



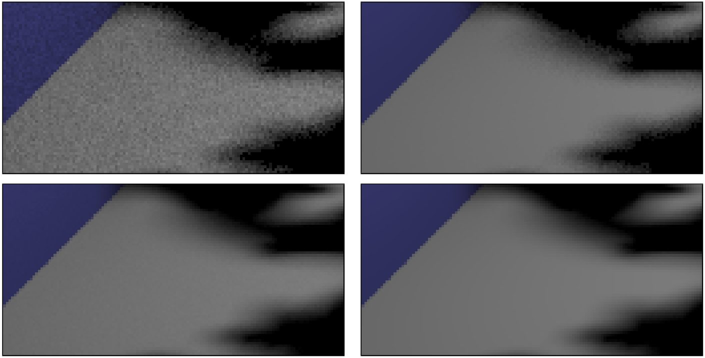
Minimum variance: Outside proxies



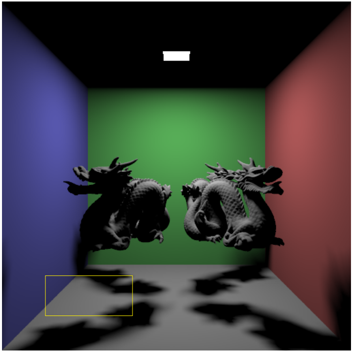


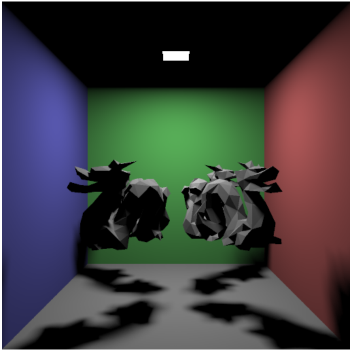
Minimum variance: Outside proxies

	Outside proxy equal probabilities	**Outside proxy** optimal probabilities
16sr		
1024sr		



Minimum variance: General proxies



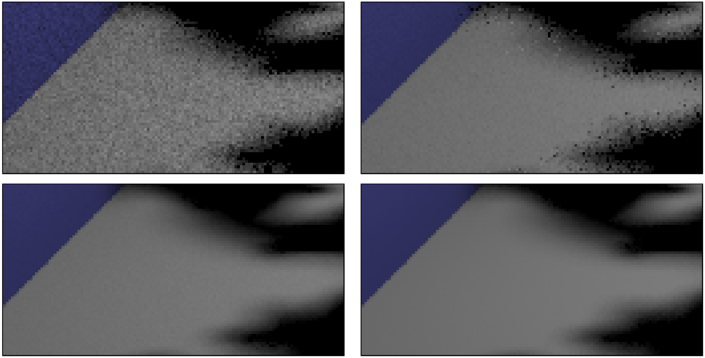


Minimum variance: General proxies

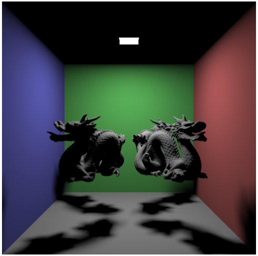
General proxy equal probabilities | **General proxy** optimal probabilities

16sr

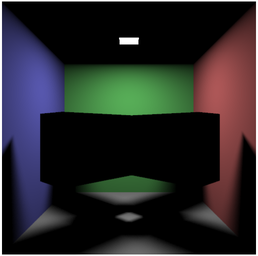
1024sr



Minimum variance: Problem

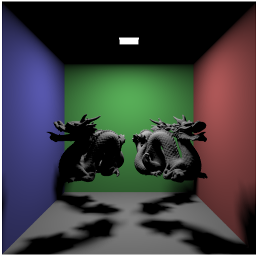


Original geometry



Outside proxies

Minimum variance: Problem



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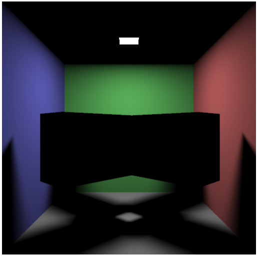
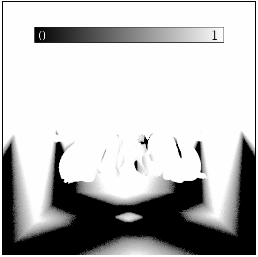
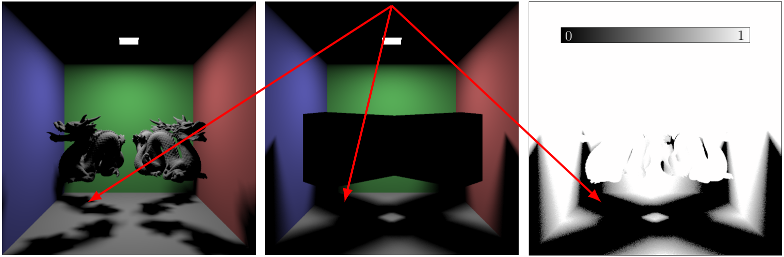


Outside proxies



Optimal p_{proxy}

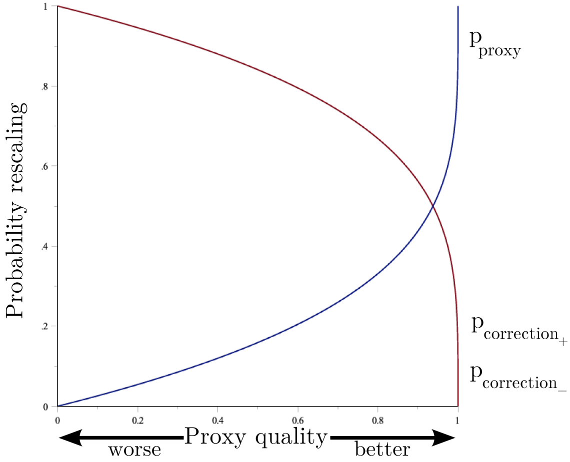
Minimum variance: Problem



Original geometry **Outside proxies** **Optimal p_{proxy}**

Variance minimization does not account for intersection cost. Therefore the term with the exact visibility will be favored in umbra regions.

Favour cheap geometry proxy tests



Favour cheap geometry proxy tests

Increase p_{proxy} when $V_P(x,y) \cong V_{P'}(x,y)$

$$error = \frac{\#\text{ probe rays where } V_P(x,y) \neq V_{P'}(x,y)}{\#\text{ probe rays}}$$

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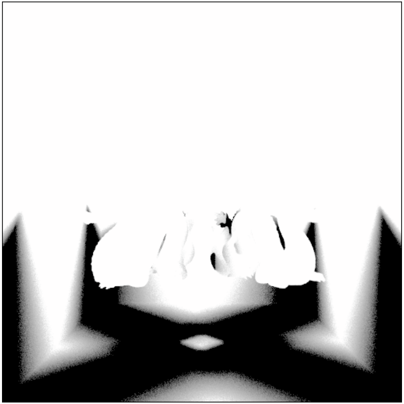
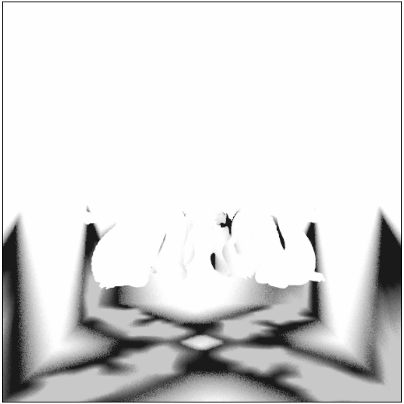
$$error = \frac{\#\text{ probe rays where } V_P(x,y) \neq V_{P'}(x,y)}{\#\text{ probe rays}}$$

$$p'_{correction_+} = \sqrt[4]{error} \cdot p_{correction_+}$$
$$p'_{correction_-} = \sqrt[4]{error} \cdot p_{correction_-}$$
$$p'_{proxy} = 1 - p'_{correction_+} - p'_{correction_-}$$

Favour cheap geometry proxy tests

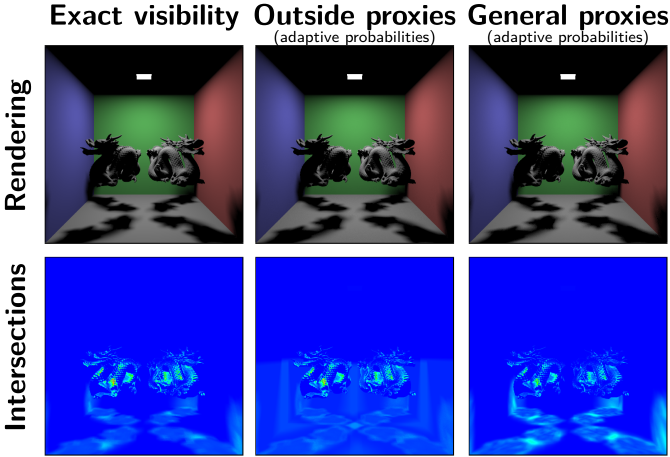


Optimal $\mathbf{p_{proxy}}$



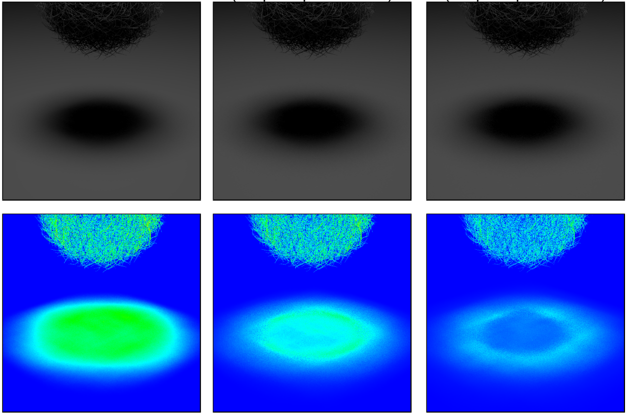
Modified $\mathbf{p_{proxy}}$

Results: Equal time



	Exact visibility	Outside proxies (adaptive probabilities)	General proxies (adaptive probabilities)
Shadow rays per shading point	256	192	240
MSE	4.461×10^{-7}	5.36×10^{-6}	2.448×10^{-6}

Results: Equal time



	Exact visibility	Outside proxies	General proxies
Shadow rays per shading point	256	352	480
MSE	6.158×10^{-7}	9.689×10^{-6}	9.916×10^{-6}

Conclusion

- Stochastic evaluation of visibility using geometry proxies
  - Unbiased images

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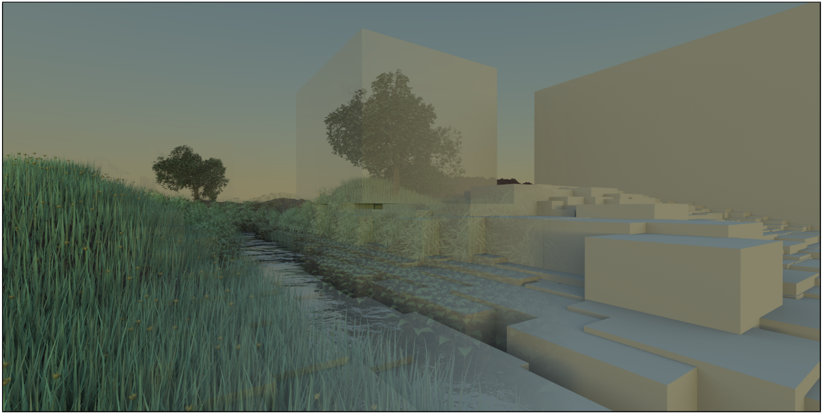
- Stochastic evaluation of visibility using geometry proxies
  - Unbiased images
- Theoretical framework
- New and experimental look at visibility
  - Hope to inspire new future work

Thank you for your attention



Outside proxies for the famous Nature scene [Pharr10]